**Review of Economics & Finance**
**Submitted on 03/10/2014**
**Article ID: 1923-7529-2015-01-01-21**
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# Empirical Estimation of the Cost of Equity: An Application to Selected Brazilian Utilities Companies[1]

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**Abstract:** We provide an extensive set of alternative models for the estimation of the real cost of equity in a sample of utilities firms in Brazil with monthly data from March 2006 to June 2011. The traditional CAPM is rejected, together with the Fama-French factors, due to a poor fit. Additional factors improve the fit of the models and the estimated betas and real cost of equity increase relative to the traditional CAPM and Fama-French models. Accounting for conditional heteroskedasticity shows that autocorrelation of variances is more important than news effects. The inclusion of higher order terms shows that the third order term is mostly significant and positive indicating preference for skewness in this sample period. Our estimates of betas and the implied predicted real cost of equity show that, across the best models, betas are significantly below unity in the range 0.26-0.73. The predicted real cost of equity, across the best models, for Brazil in this sector and sample period is in the range of 8.7% to 13.2% per year.

**Keywords**: Asset pricing, Multifactor model, CAPM, Emerging markets

**JEL Classifications**: G12, G31, O54

## 1. Introduction

Brazil is one of the emerging economies that have attracted attention of investors and policymakers worldwide in the last ten years or so due to its continuous economic growth. Economic growth is also accompanied with an increased role of institutions that regulate and supervise the private companies that provide public utilities and services to the population. When private companies decide to invest in long-term public utility projects, the main objective is to maximize shareholder value. The need for return must be weighted by the risk of the project and the framework of the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966) has proven useful for this kind of analysis.

We undertake the challenge of providing a broad set of empirical evidence on the risk faced by Brazilian companies and the implied predicted real cost of equity with focus on public utility

[1] We thank the Department of Economics at FEA-USP, FIPE, Fapesp and the Department of Economics at Tufts University for financial support.

companies in the Brazilian capital market in the 2006-2011 period. Our focus on the public utilities sector derives from the critical role it plays in providing infrastructure for economic activity. We estimate conditional variants of the CAPM and account for conditional heteroskedasticity, alternative distributional assumptions and the inclusion of higher order terms looking for risk aversion and preference for skewness. In particular, we provide estimations for a sample of 26 Brazilian public firms in the utilities sector. Those firms are potential beneficiaries of public concessions and an accurate estimation of the cost of capital is important for the success of the enterprise and ultimately the welfare of individuals.[2]

We find that the traditional CAPM and the Fama-French three factor model are a poor fit and give low betas and low estimated real return on equity. Additional factors improve the fit of the models, the estimated betas increase relative to the traditional and Fama-French CAPM, while the estimated real cost of equity also increases. Our time sample period includes the U.S. financial crisis of 2008 and we observe marked conditional volatility in this period. We extend our econometric estimates to include variants of the conditional heteroskedasticity model of Engle (1982) and find that they influence significantly the estimation of the cost of equity in Brazil in this period. Accounting for conditional heteroskedasticity shows that autocorrelation of variances is more important than news effects and, under the fatter tailed student's *t* distribution, the estimated betas are slightly lower and the estimated real returns are lower than under normality. Since deviations from normality are critical in the estimation of the CAPM, we use also the generalized method of moments (GMM). GMM models give a higher beta and a higher estimated real cost of equity when all factors are included, but the estimated betas are less precise in some cases.

Our alternative model estimates provide a sample of betas and predicted real cost of equity that is used to give a range of plausible betas and predicted real cost of equity for Brazil in the last six years. Our average estimates of the betas and the real cost of equity across all estimated models show that beta for this sector is well under unity and in the range of 0.33-0.61; and predicted real cost of equity in the range of 4.5% to 11.5%. However, several of those models are rejected and/or give a poor fit. When we focus on the four best models under the criteria of least residual sum of squares, our average estimates of the betas is well under unity, but in a slightly wider range of 0.26-0.73. This indicates that firms in this sector respond to changes in the Brazilian market premium by only a one to three-fourths change in their own premium. For the best models, the predicted real cost of equity for Brazil in this sector and sample period is in the range of 8.7% to 13.2% averaging approximately 11% so that, from a public policy perspective, values within this range should be deemed plausible.

The remainder of the paper is as follows. Section 2 provides a literature review. Section 3 describes the methodology and section 4 describes the data. Section 5 is the core econometric analysis of the estimation of the real cost of equity via multifactor models. Section 6 provides an analysis of the alternative econometric models. Section 7 concludes and the appendixes provide additional statistical tables and tests.

## 2. Literature Review

According to the classic capital asset pricing model of Sharpe (1964), Lintner (1965) and Mossin (1966) the premium of equity is proportional to the market premium. Early empirical tests of the CAPM by Lintner (1965), Black, Jensen and Scholes (1972), Fama and MacBeth (1973, 1974) provided evidence of rejection of the model. Fama and MacBeth (1973, 1974), and Fama and

[2] Bianconi, Yoshino and Sousa (2013) present an analysis of Brazilian financial assets in a comparative context of the BRIC and the influence of the U.S. financial crisis.

French (1993, 2004) provided convincing evidence that additional factors should be included for an empirical explanation of the equity premium. In the Brazilian equity market, Yoshino and Bastos-Santos (2009) provide an important benchmark for the traditional versus expanded CAPM, showing that the empirical evidence is in favor of expanded models.[3] Bianconi and Yoshino (2012) expand on this framework examining the market performance of real estate companies in Brazil from the perspective of general multifactor models. Our basic approach is to use panel data methods. Barnes and Hughes (2001) focus on the panel nature of financial data and find that the panel approach for the CAPM yields more precise parameter estimates and greater understanding of the significance of both conditional variables and multi-factors. Cheng, Lai, and Lu (2005) use panel methods for a sample of Taiwanese firms and find that they lead to more explanatory power than the traditional OLS methods.

Other common issue with the CAPM is its reliance on normality assumptions. One common response to the evidence of non-normality is to use GMM, which greatly relaxes distributional assumptions relative to OLS. We extend the panel approach via GMM estimation here. For example, Vorkink (2003) compares OLS and GMM based tests of asset pricing models and fails to reject the CAPM. On another dimension, Merton (1980) shows that in estimating CAPM models of the expected market return, estimators which use realized returns should be adjusted for hetero-scedasticity. Our time sample period includes the U.S. financial crisis of 2008 and we include variants of the conditional heteroskedasticity model of Engle (1982) and find that they influence significantly the estimation of the cost of equity in Brazil in this period. Tsuji (2009) evaluates the conditional asset pricing models for the Japanese stock market using a multivariate generalized autoregressive conditional heteroskedasticity panel (GARCH) model. The evidence shows that the CAPM can be represented by using the multivariate GARCH model to explain the value premium in Japan.[4]

In addition, other nonlinearities are accounted for in our econometric models. In the classic mean-variance framework, Kraus and Litzenberger (1976) examine higher order effects. More recently, Piccioni (2012) shows that the CAPM framework can potentially allow for higher order effects of risk via approximation of utility with asymmetric polynomial models that allow for different attitudes toward risk in the domains of gains and losses. Our research is also intimately related to the literature on capital budgeting for Latin America. One World Bank perspective is presented in Sirtaine et al. (2004) who estimate the returns that private investors obtain in infrastructure projects in Latin America. Their findings are that the financial returns of private infrastructure concessions have been modest, but that the variance of returns across concessions and countries is considerably large. Grandes et al. (2006) test the Fama and French three factors model for a sample of 921 Latin American stocks over 1986-2004 and find that it does not add significant explanatory power to CAPM regressions of Latin American equity premiums.

Specifically for Brazil, Sanvicente (2011) advocates the use of the opportunity cost of capital for capital budgeting via the CAPM. Villarreal and Cordoba (2010) present a methodological approach for the calculation of the discount rate in emerging markets requiring a robust estimation of country risk and also estimation of the cost of equity via CAPM. More generally, in the U.S.,

[3] See Yoshino (2003) for earlier evidence on risk and volatility in the Brazilian stock market; and Estrada (2007) for four case studies in emerging markets showing that the assumptions behind the CAPM are compromised when applied to a country where the financial and institutional environment show more volatility relative to a developed one.

[4] Pesaran and Yamagata (2012) suggest that abnormal profits are earned during episodes of crisis and market inefficiencies.

Graham and Harvey (2001) find that 74% of their survey firms use the CAPM for capital budgeting. For the case of highway concessions, Goldberg (2009) compares valuations of owners and operators of toll roads in Australia and the U.S. He shows that potential overvaluation increases the risk of insolvency through excessive borrowing.[5] More recently, Kruger et al. (2011) show that the use of a unique company-wide discount rate in investment may lead to overinvestment (underinvestment) when a market beta is higher (lower) than the industry beta whereas Jagannathan et al. (2011) find that firms behave as though they add a hurdle premium to their CAPM based cost of capital.

Overall, finance theory restrictions on the behavior of valuation and the links of observed stock prices to the level of the equity premium have proven useful for the estimation and prediction of equity premiums. The literature suggests that using multifactor models for the estimation of the real cost of equity is common and provides useful benchmarks. Here, we propose an extensive set of alternative multifactor models using multiples, risk factors and potential U.S. factors for the estimation of the real cost of equity for utilities firms in Brazil, an emerging market, and use robust statistical methods to support the predictive validity of the models proposed. We use this approach here for Brazilian utility public companies.[6]

## 3. Methodology

The standard CAPM shows that the expected return of equity *i* is related to the expected market premium as

$$E[r_{ei}] = r_f + \beta_{ei}(E[r_M] - r_f) \tag{1}$$

where $E[r_{ei}]$ is the expected equity nominal return of company $i$ , $r_f$ is the risk free nominal interest rate, $E[r_M]$is the expected nominal return on the market and $\beta_{ei} = \frac{cov(r_{ei}, r_M)}{var(r_M)}$ is the beta of the equity. This paper focuses on estimating forms of the CAPM empirically to obtain estimates of the return on equity. The standard CAPM can be estimated, using ex-post observed data, as

$$r_e - r_f = \alpha_0 + \alpha_1\{r_M - r_f\} + u \tag{2}$$

thus, it uncovers $\alpha_1$as an estimate of the beta. We can estimate equation (1) using panel data fixed firm and time effects as

$$r_{eit} - r_{ft} = \alpha_0 + \alpha_1\{r_{Mt} - r_{ft}\} + \gamma_{1i} + \gamma_{2t} + u_{it} \tag{3}$$

for firm *i* and period *t* where the CAPM needs the restriction that $\alpha_0 = 0$. The CAPM with additional factors can be similarly estimated, using ex-post observed data, as

$$r_e - r_f = \alpha_0 + \alpha_1\{r_M - r_f\} + \sum_{j=1}^{k} \delta_j X_j + u \tag{4}$$

[5] Alexander et al. (2001) finds that an information gap is especially important in determining the degree of market risk, a critical component of the cost of capital demanded by operators. Vassallo (2010) examines a mechanism of awarding the concession to the bidder who offers the least present value of the revenues discounted at a discount rate fixed by the government in the contract based on Engel et al. (2001). He finds that the lower the discount rate, the larger will be the traffic risk allocated to the concession.

[6] It is well known that there are several alternative models for cost of equity estimation. Kolouchová and Novák (2010) present a survey of approaches commonly used in the U.S. and Europe and they include CAPM, CAPM including some other risk, APT, market return adjusted for risk, average historical return, dividend discount model, investor expectations, regulatory decisions, E/P ratio, and cost of debt plus risk premium for equity. Our objective here is to use the multivariate factors approach using the enriched framework of panel data methods.

where $\alpha_1$is an estimate of the beta and $\sum_{j=1}^{k} \delta_j X_j$ represents potential additional factors such as the ones introduced by Fama and French and others. We can estimate equation (3) using panel data fixed firm and time effects as

$$r_{eit} - r_{ft} = \alpha_0 + \alpha_1\{r_{Mt} - r_{ft}\} + \sum_{j=1}^{k} \delta_j X_{jit} + \gamma_{1i} + \gamma_{2t} + u_{it} \tag{5}$$

for firm i and period t where the CAPM needs the restriction that $\alpha_0 = 0$.[7] We further estimate equation (5) to include the family of ARCH models of volatility as well as via GMM and including nonlinearities reflecting risk aversion and preference for skewness. The family of generalized autoregressive conditional heteroskedastic models considered here has the basic form

$$r_{eit} - r_{ft} = \alpha_0 + \alpha_1\{r_{Mt} - r_{ft}\} + \sum_{j=1}^{k} \delta_j X_{jit} + \gamma_{1i} + \gamma_{2t} + \psi \quad h_{it-1} \quad + u_{it} \tag{6a}$$

$$h_{it} = \xi_0 + \xi_1 {u_{it-1}}^2 + \xi_2 h_{it-1} \tag{6b}$$

for firm *i* and period *t* where the CAPM needs the restriction that $\alpha_0 = 0$ and $h_{it}$ denotes the conditional variance of $u_{it}$, $Var_{t-1}(u_{it})$. This specification allows ARCH-in-mean when $\psi \neq 0$ and GARCH when $\xi_2 \neq 0$. ARCH effects are the effects of innovations (news) in the skedastic function when $\xi_1 \neq 0$. Garch when $\xi_2 \neq 0$ reflects autocorrelation in the skedastic function. Estimation is by standard maximum likelihood methods. We also estimate models (8a-b) with the assumption that the errors have a student's *t* distribution, i.e. slight fatter tails. We estimate models more generally via GMM with instrumentation for lagged dependent variable and heteroskedasticity and autocorrelation (hac) robust standard errors. The basic GMM econometric model is

$$E[\,(r_{eit} - r_{ft}) - \alpha_0 - \alpha_1\{r_{Mt} - r_{ft}\} - \sum_{j=1}^{k} \delta_j X_{jit} - \gamma_{1i} - \gamma_{2t}\,] = 0 \tag{7}$$

In the case of nonlinear effects, it refers to appending all models discussed above to include quadratic and cubic effects of the market premium, or

$$r_{eit} - r_{ft} = \alpha_0 + \alpha_1\{r_{Mt} - r_{ft}\} + \alpha_2\{r_{Mt} - r_{ft}\}^2 + \alpha_3\{r_{Mt} - r_{ft}\}^3 + \cdots \tag{8}$$

This specification allows preference towards risk sensitivity when $\alpha_2 \neq 0$, and preference for skewness when $\alpha_3 \neq 0$; see e.g. Harvey (2000), Piccioni (2012) and several of the references therein.[8] In particular, if $\alpha_2 > 0$ it implies that the firm premium increases (decreases) as the "variance" of the market premium increases (decreases). In the case where $\alpha_3 > 0$ it implies that the firm premium increases (decreases) as the positive (negative) skewness of the market becomes larger. Finally, using the predicted premium from the estimated models above, we obtain an estimate of the real equity cost per year using the formula

$$\{1 + [\left(\frac{1+\{E[\,(r_{eit}-r_{ft})|.]+r_{ft}\}}{1+\pi}\right) - 1]\}^{12} - 1 \tag{9}$$

where $E\left[\,(r_{eit} - r_{ft})|.\right]$ is the predicted premium from the estimated model and $\pi$ is the consumer price inflation rate of the IPCA. Hence, we just add the nominal measure of the long term interest rate and discount for the consumer price inflation IPCA; then we obtain the estimate of the monthly predicted real return on equity for the sector which is then annualized.

[7] The standard definition of factor models usually requires that the factors are common amongst the individual assets; however we do allow for firm-specific factors in our models since they provide an important source of idiosyncratic risk potentially not diversified in the Brazilian market.

[8] This form of the CAPM is also sometimes called the Four Moment CAPM; see e.g. Harvey (2000).

## 4. **Data**

We use monthly data of 26 regulated utilities firms in Brazil in the period March 2006 to June 2011.[9] On average, they represent 13.1% of the total market value in the sampled period. The variables used in the analysis are the following. For the traditional CAPM, Premium_co is the premium of the firms defined as the monthly return of the firm stock listed on the Sao Paulo Bovespa stock exchange minus the 5 year Idka risk-free interest rate[10]. This reflects a premium over a moderately long term interest rate given that highway concessions are long term investments. Premium_mkt is defined as the monthly return of the Sao Paulo Bovespa stock index minus the 5 year Idka risk-free interest rate which represents the market premium. The Fama-French additional factors are Book_to_mkt defined as the book value divided by the market value of the firm and Mkt_Value is the (logarithm) of the market value of the firm.

In the multiples category we have P_Profit defined as market price of the shares over company's profit for the period. Price_TotAssets in the category of patrimonial value is price over the total assets of the company. Price_Sales in the category of revenues is defined as price/sales ratio. Fixed_Total is defined as fixed assets over total assets measuring the fixed immobilized share of the value of the company. Liquidity_curr is current liquidity measured as the ratio of current assets over current liabilities reflecting the capacity to pay of the company in the short term. Debt_Equity is the ratio of debt to equity. Ev_Ebitda is in the category of enterprise value defined as EV-Enterprise Value (Market value of the company added to its net debt) over the EBITDA (earnings before interest, taxes, depreciation and amortization). Leverage_Fin is financial leverage. Companies that are highly leveraged may have a greater risk of bankruptcy if they are unable to make debt service payments (financial distress) in an adverse situation, as in the recent phenomenon of the credit crunch. Highly leveraged companies may not find new lenders in the future. On the other hand, the financial leverage can have benefits to increase the return of shareholders because of the tax advantages associated with loans (tax shield). As part of macroeconomic factors we have Cred_GDP defined as the ratio of total credit to GDP in Brazil. Exch_rate is the nominal exchange rate of the Brazilian Real vis-à-vis the US dollar. Vix is the Chicago Board Options Exchange Market Volatility Index (VIX), a risk measure implicit in options on the U.S. stock market S&P500 index. Known as the index of "fear," this index represents a measure of expectation of stock market volatility over the next 30 days. Sharpe_Nasdaq is the Sharpe ratio of the Nasdaq index in the U.S., measuring excess return of technology stocks per unit of risk. Case_S is the Case_Shiller house price index in the U.S., capturing variations in the U.S. housing market. Finally, Fin_crisis is a dummy variable taking unity value on and after September 2008 as a proxy for the U.S. financial crisis. Table 1 presents the summary statistics of the variables.

## 5. Empirical Models for the Real Cost of Equity

In this section we present the basic alternative estimations and results of our econometric models for the real cost of equity.

### 5.1 CAPM and Fama-French Models

The estimated beta in the classic CAPM from the panel data with firm fixed effects only as in expression (3) is in Table 2. The estimated beta is 0.66, well below one and the portfolio of

[9] The firms in the sample are described in the appendix available at our website: http://www.tufts.edu/~mbiancon/research.html

[10] Idka is a set of indices that measure the performance of synthetic federal public bond portfolios with constant maturities, in this case the 5 year maturity.

companies is less volatile than the market. However, note that the restriction that the constant is zero is not satisfied in this case and the classic CAPM is rejected, a result common in the literature in the U.S. since Black et al. (1972), and shown by Yoshino and Bastos-Santos (2009) for the case of the Brazilian market. We then follow up by estimating the real return on equity from the model according to expression (9) or using the predicted premium from the regression, subtracting the nominal measure of the long term interest rate and discounting for the consumer price inflation IPCA and annualizing. In this case, the estimated return on equity is 3.62% per year in real terms, shown in the last rows of Table 2. Figure 1 shows the predicted versus actual real returns and the security market space. In panel a., the red line is the 45° and a good fit would have the data along the line.[11]

**Table 1.** Variables in the Sample and Descriptive Statistics

| Variable | Mean | Median | StdDev | Skewness | Kurtosis | N |
|---|---|---|---|---|---|---|
| Premium_co | 0.00 | 0.00 | 0.12 | -0.83 | 9.07 | 1495 |
| Premium_mkt | 0.00 | 0.00 | 0.07 | -0.73 | 4.91 | 1612 |
| Book_to_mkt | 0.79 | 0.65 | 0.61 | 2.28 | 12.52 | 1462 |
| Mkt_Value | 22.34 | 22.57 | 1.19 | -0.91 | 3.42 | 1506 |
| P_Profit | 32.73 | 15.00 | 568.98 | -0.68 | 109.24 | 1320 |
| Price_TotAssets | 2.46 | 1.80 | 2.22 | 2.28 | 9.33 | 1386 |
| Price_Sales | 3.48 | 3.10 | 2.66 | 2.10 | 11.16 | 1320 |
| Ev_Ebitda | 1.99 | 8.10 | 237.76 | -18.34 | 342.82 | 1067 |
| Vix | 23.54 | 22.05 | 10.49 | 1.44 | 5.11 | 1612 |
| Exch_rate | 1.89 | 1.82 | 0.23 | 0.51 | 2.08 | 1612 |
| Cred_GDP | 38.28 | 38.90 | 6.27 | -0.11 | 1.45 | 1612 |
| Case_S | 166.36 | 154.50 | 25.59 | 0.47 | 1.52 | 1612 |
| Sharpe_Nasdaq | 0.02 | 0.28 | 1.08 | -0.79 | 3.09 | 1612 |
| Fin_crisis | 0.53 | 1.00 | 0.50 | -0.13 | 1.02 | 1612 |
| Fixed_Total | 124.15 | 114.90 | 108.71 | 4.79 | 41.91 | 1565 |
| Leverage_Fin | 58.20 | 1.90 | 1266.23 | 22.69 | 515.85 | 1555 |
| Liquidity_curr | 1.19 | 1.10 | 0.48 | 0.80 | 4.51 | 1565 |
| Debt_Equity | 0.61 | 0.44 | 0.67 | 3.48 | 19.76 | 1499 |

**Legends (These legends apply to the tables and figures throughout this paper):**

Premium_co = monthly return of the firm stock listed on the Sao Paulo Bovespa stock exchange minus the 5 year Idka risk-free interest rate

Premium_mkt = monthly return of the Sao Paulo Bovespa stock index minus the 5 year Idka risk-free interest rate which represents the market premium.

Book_to_mkt = book value divided by the market value of the firm

Mkt_Value =logarithm of the market value of the firm.

P_Profit = market price of the shares on the company's profit for the period

Price_TotAssets= price over the total assets of the company

Price_Sales= sales over price

Fixed_Total= fixed assets over total assets

Liquidity_curr= ratio of current assets to current liabilities

Debt_Equity= debt to equity

[11] See Cochrane (2005) for details of those plots in the context of the CAPM.

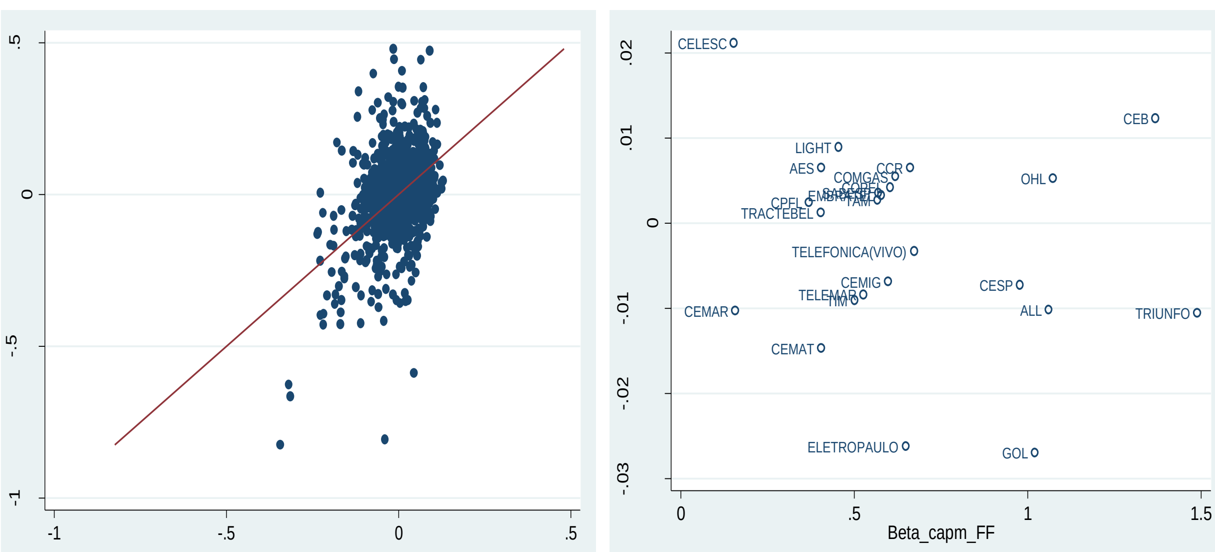
Ev_Ebitda=Market value of the company added to its net debt over earnings before interest, taxes, depreciation and amortization
Leverage_Fin= financial leverage
Cred_GDP= ratio of total credit to GDP in Brazil
Exch_rate= exchange rate of the Brazilian Real vis- à-vis the US dollar; R$/US
Vix =volatility of options on the stock market S&P500 index
Sharpe_Nasdaq= excess return of technology stocks per unit of risk
Case_S= Case_Shiller house price index in the United States
Fin_crisis= dummy variable taking unity value on and after September 2008

**Table 2.** Traditional CAPM (Firm F.E.)

**Note:** *** indicates significance p<0.001

| | **(1) premium_co** |
|---|---|
| premium_mkt | 0.660*** |
| constant | -0.00264*** |
| Sample size | 1495 |
| adj. $R^2$ | 0.147 |
| Estimated return on equity in real terms | 3.62% |

This is not the case in this model and the fit is poor. Panel b. shows the relationship between the expected premium of the company and the beta of the company. The security market line would be a fitted line that should have a positive slope from the origin where the return is the risk free return. However, this sample period includes the financial crisis and reflects a tendency for a potential security market line that is negatively sloped on average. The potential realized losses indicate an appetite for risk ex-post, see Merton (1980).[12] The figure also shows that a group of firms is well above the origin line and would be considered healthy from the perspective of risk-averse investors. Those include companies CCR, OHL, and CEB among others. Another group is well below the origin and would indicate preference for risk by investors. They include companies CEMIG, CESP, and GOL among others.




**a.** Predicted versus Actual Real Returns **b.** Predicted Premium versus Beta

**Figure 1.** CAPM – Fama French Factors

[12] Associated with each table discussed in the text we have an additional appendix with tables of the actual and predicted moments of the monthly real returns for the models in the table; tests for autocorrelation in the panel for the predicted real returns; and normality tests for the panel and by firm. Those are available upon request.

Next in Table 3, we have the estimated beta in the classic CAPM from the panel data with firm fixed effects only as in expression (3). The estimated beta is 0.682, so that the portfolio of companies is also less volatile than the market. The restriction that the constant is zero is satisfied in this case. Similarly, from expression (9) using the predicted premium from the regression, subtracting the nominal measure of the long term interest rate and discounting for the consumer price inflation IPCA, we obtain an estimate of the monthly predicted real return on equity for the sector. In this case, the annualized estimated real return on equity is 3.36% per year in real terms. The predicted versus actual real returns gave a marginal improvement in fit from the traditional CAPM, the fit continues to be poor. We re-estimate the two models above including year fixed effects in addition to firm fixed effects. The estimated beta in the classic CAPM goes from 0.660 to 0.623 and in the Fama-French models from 0.682 to 0.607, well below one and the portfolio of companies continues to be less volatile than the market. The estimated real return on equity goes from 3.62% to 3.92% in the traditional CAPM and from 3.36% to 3.47% in the Fama-French model per year in real terms.[13]

**Table 3.** Fama-French Factors (Firm F.E.)

| | **(1) premium_co** |
|---|---|
| premium_mkt | 0.682*** |
| Mkt_Value | 0.0157 |
| book_to_mkt | -0.0395* |
| constant | -0.324 |
| Sample size | 1345 |
| adj. $R^2$ | 0.180 |
| Estimated return on equity in real terms | 3.36% |

**Note:** * and *** indicate p<0.05 and p<0.001, respectively

### 5.2 Additional Factor Models

Next, we add several additional factors to the estimation of the real cost of equity. First, we select multiples representing the various characteristics of the company. Regarding profit, we select the variable market price of shares to the company's earnings for the period: price-to-earnings ratio. In the category of book value, we include the quotation of the shares on the stock exchange over the equity value of the company in the period: Price/book value (P/V). In the category of revenues, we have included the quotation of the shares on the stock exchange over sales in the period: Price/sales. In the category of enterprise value, we include the ratio EV/EBITDA and as another variable that is specific to the company, we have included financial leverage. Among other common risk factors, we included the real exchange rate of the Brazilian Real vis-à-vis the U.S. dollar and the credit/GDP relation in Brazil. We also included the S&P500 volatility index, the VIX (Chicago Board Options Exchange Market Volatility Index) and the Sharpe ratio of the Nasdaq in the U.S. Regarding the financial crisis, we include the Case_Shiller U.S. housing market index and a dummy variable about the September 2008 financial crisis.

Including only firm fixed effects Beta continues in the 0.6 range, the estimated return on equity is an order of magnitude higher, 9.56% per year in real terms and the restriction that the constant is zero is satisfied in this case. Several additional factors are significant including the price/sales ratio, the EV/EBITDA ratio, the VIX volatility index and financial leverage. The predicted versus actual real returns and the fit of the model improves relative to the other models without the additional factors. The same regression but including firm and year fixed effects. Beta continues in the 0.6 range, the estimated real return on equity is 9.68% per year in real terms and the restriction that the constant is zero is satisfied in this case. The inclusion of year fixed effects leaves price/sales and financial leverage statistically significant. The predicted versus actual real returns. There is a significant improvement relative to the previous models in terms of fit.

[13] The predicted premium versus beta figures are similar to the Figures 1b and 2b and are available upon request for all subsequent models.

We also restrict the set of factors to Book-to Market; Price/sales; EV/EBITDA; and financial leverage since those are the ones that significantly impact on the premium. With firm fixed effects only we obtain beta of 0.603 and estimated return on equity of 9.85% per year in real terms. In this case the restriction that the constant is zero is satisfied. We also included year fixed effects. The coefficient of EV/EBITDA is statistically zero (left out of the table); beta declines to 0.541 and estimated return declines to 6.94% per year in real terms.

Overall, the evidence is that the additional factors improve the fit of the models. The estimated betas increase relative to the traditional CAPM and Fama-French, while the estimated real cost of equity also increases.

### 5.3 Conditional Heteroskedasticity

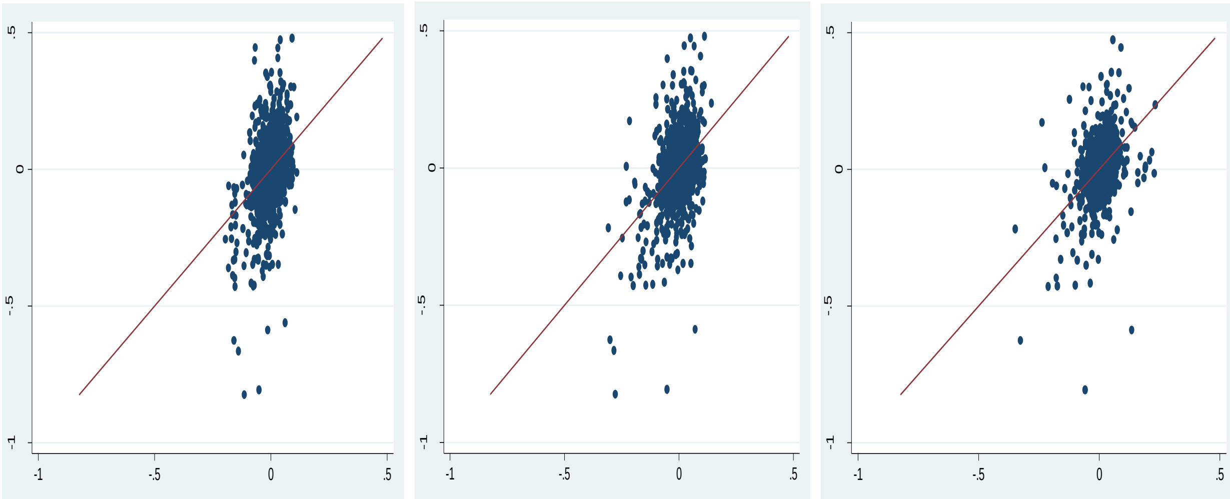
The sample period of analysis includes the U.S. financial crisis of 2008 and there is evident time varying volatility in this period all over the world. Thus, we include the potential for conditional heteroskedasticity in our panel estimates according to the family of generalized autoregressive conditional heteroskedastic models in expressions (6a-b). In particular, arch effects are the effects of innovations (news) in the skedastic function when $\xi_1 \neq 0$ and garch when $\xi_2 \neq 0$ reflects autocorrelation in the skedastic function. We estimate models (6a-b) under the alternative assumptions of normality and student's *t* distribution.

**Table 4**. Arch Family Firm and Year F.E.

**Note:** *, **, and *** indicate p<0.05, p<0.01, and p<0.001, respectively.

| | (1) premium_co | (2) premium_co | (3) premium_co |
|---|---|---|---|
| **premium_mkt** | 0.544*** | 0.505*** | 0.458*** |
| **Mkt_Value** | | 0.0228 | -0.000599 |
| **book_to_mkt** | | -0.0702*** | -0.0831 |
| **Price_Sales** | | | 0.0114 |
| **Ev_Ebitda** | | | -0.000391 |
| **Leverage_Fin** | | | -0.0000162 |
| **constant** | 0.0317** | -0.277 | 0.0606 |
| **ARCHM sigma2** | -0.531 | 0.966 | 1.335* |
| **ARCH_ L.arch** | 0.138* | 0.146* | 0.189* |
| **ARCH_ L.garch** | 0.814*** | 0.826*** | 0.779*** |
| **ARCH_cons** | 0.000498* | 0.000253 | 0.000214 |
| **Estimated return on equity in real terms** | 6.93% | 8.77% | 13.96% |
| **Sample size** | N=1495 | N=1345 | N=885 |

First, Table 4 shows models from the traditional CAPM (first column), the Fama-French factors (second column) and multiples with firm and year fixed effects. In the traditional CAPM, column (1), the beta is 0.544, an order of magnitude less than the previous models, and the constant is statistically different than zero thus rejecting the basic CAPM. The arch and garch components of equation (6b) are significant and the garch term is larger in magnitude than the arch term indicating that autocorrelation of the variances is more important than the news term. The arch in mean term in the main equation (6a) is not significant. The estimated real return on equity in this model is 6.93% per year. In the CAPM with the Fama-French factors in column (2), the beta declines to 0.505, less than the previous models, but the constant is statistically zero. The arch and garch components of equation (6b) are significant and the garch term is larger in magnitude than the arch term indicating that autocorrelation of the variances is more important than the news term. The arch in mean term in the main equation (6a) is not significant. The estimated real return on equity in this model increases to 6.77% per year. In the CAPM with the Fama-French factors and the additional factors in column (3), the beta declines further to 0.458 and the constant is statistically zero. The factors are not statistically significant. The arch and garch components of equation (6b) are significant and the garch term is larger in magnitude than the arch term. The arch in mean term in the main equation (6a) is not significant. The estimated real return on equity in this model increases even further to 13.96% per year. Figure 2 shows the predicted versus actual real returns. It is instructive to note that as more factors are added the fit of the model improves, and the model of column (3) is a better fit.




**a.** Model (1) **b.** Model (2) **c.** Model (3)

**Figure 2** Predicted versus Actual Real Returns

A common issue with financial markets data is the possibility of deviations from normality, or fatter tails.[14] We estimate models from the traditional CAPM, the Fama-French factors and with additional multiples with firm and year fixed effects, but with the assumption that the errors have a student's *t* distribution, i.e. slight fatter tails. In the traditional CAPM, Beta falls to 0.460 and the constant is statistically different than zero thus rejecting the basic CAPM. The arch and garch components of equation (6b) are significant and the garch term is larger in magnitude than the arch term indicating that autocorrelation of the variances is more important than the news term. The arch in mean term in the main equation (6a) is not significant. The estimated real return on equity in this model is 5.43% per year, lower than the 6.93% per year under normality. In the CAPM with the Fama-French factors, Beta declines only slightly to 0.443, less but the constant is statistically zero. The arch and garch components of equation (6b) are significant, the garch term is larger in magnitude than the arch term indicating that autocorrelation of the variances is more important than the news term. The arch in mean term in the main equation (6a) is not significant. The estimated real return on equity in this model decreases to 4.36% well below the 6.77% per year under normality. In the CAPM with the Fama-French factors and the additional factors, Beta declines further to 0.380 and the constant is statistically zero. The arch and garch components of equation (6b) are significant and the garch term is larger in magnitude than the arch term. The arch in mean term in the main equation (8a) is not significant. The estimated real return on equity in this model increases even further to 11.23%, but below the 13.96% per year under normality. Overall, conditional heteroskedasticity effects are significant and autocorrelation of variances is more important than news effects. Under student's *t* distribution, the estimated betas are slightly lower and the estimated real returns are lower than under normality.

[14] We performed normality tests of Doornik and Hansen (1994) for the panel and for each firm separately available in the web appendix. In the panel, we reject the null of normality. By firm, the great majority of firms deviate from the normal distribution.

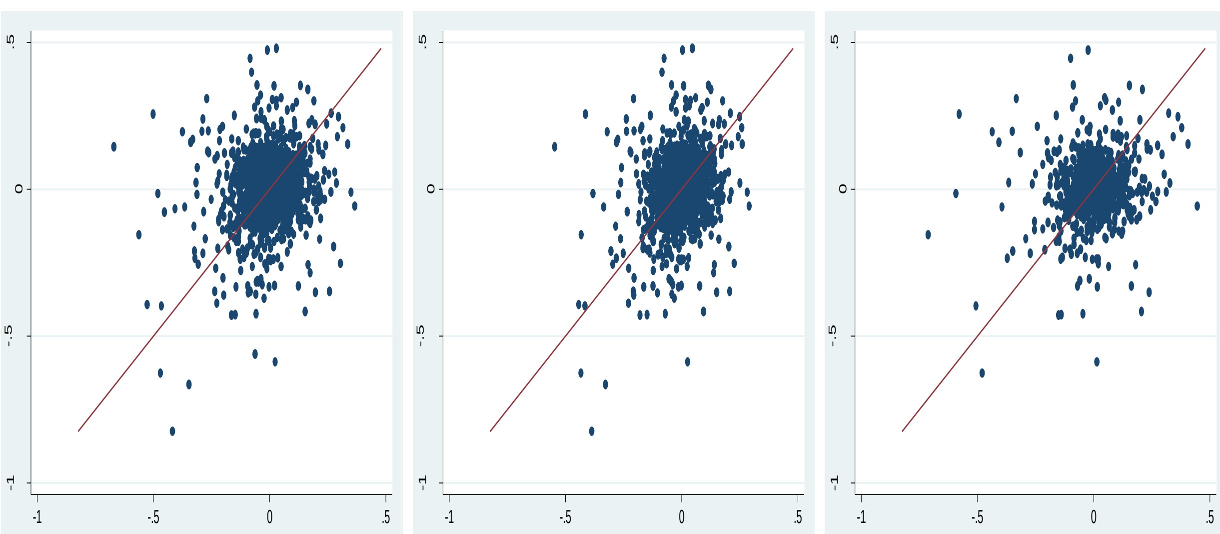
## 5.4 GMM Estimation

**Table 5**. GMM –Heteroskedasticity and Autocorrelation Robust Firm and Year F.E.

**Note:** *, **, and *** indicate p<0.05, p<0.01, and p<0.001, respectively.

| | **(1)** **premium_co** | **(2)** **premium_co** | **(3)** **premium_co** |
|---|---|---|---|
| **L.premium_co** | 0.798 | 0.630 | 0.970 |
| **premium_mkt** | 0.556 | 0.599*** | 0.509*** |
| **Mkt_Value** | | -0.00305 | -0.0142 |
| **book_to_mkt** | | -0.00749 | 0.0189 |
| **Price_Sales** | | | 0.00302 |
| **Ev_Ebitda** | | | -0.000241 |
| **Leverage_Fin** | | | 0.00000471 |
| **constant** | 0.0140 | 0.0671 | 0.156 |
| **Estimated return on equity in real terms** | 5.20% | 5.90% | 12.45% |
| **Sample size** | N=1405 | N=1264 | N=841 |

The assumption of normality is recognized as potentially unappealing for financial data. In section 5.3 above, we estimated arch models with the student's *t* distribution. In this section we estimate the models more generally via GMM with instrumentation for lagged dependent variable and heteroskedasticity and autocorrelation (hac) robust standard errors as per expression (7). Table 5 shows models from the traditional CAPM (first column), the Fama-French factors (second column) and additional multiples with firm and year fixed effects, but with GMM robust to heteroskedasticity and autocorrelation (hac). The traditional CAPM in column (1) is a poor fit since beta is not statically significant. In the CAPM with the Fama-French factors in column (2), the beta declines is of a higher magnitude, 0.6, and the constant is statistically zero. The estimated real return on equity in this model is 5.90% . In the CAPM with the Fama-French factors and the additional factors in column (3), the beta declines to 0.509 and the constant is statistically zero. The estimated real return on equity in this model increases to 12.45%. Figure 3 shows the predicted versus actual real returns. In this case, the fit is overall better than the conditional heteroskedasticity cases. Estimation via GMM gives results in terms of betas and estimated real equity returns close to the case of arch effects under the student's *t* distribution. We pursue further models via GMM below as well.




**a.** Model (1) **b.** Model (2) **c.** Model (3)

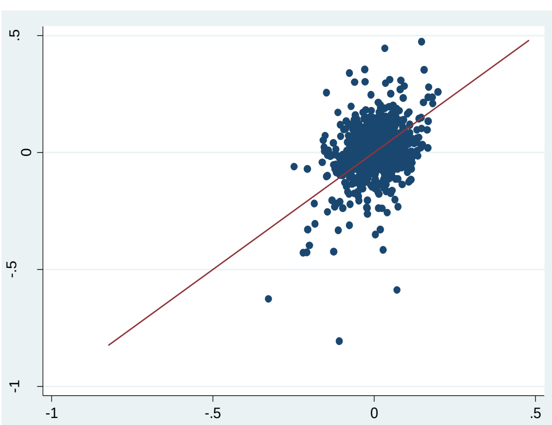
**Figure 3** Predicted versus Actual Real Returns

## 5.5 Additional Factors: Best model via Residual Sum of Squares Criteria

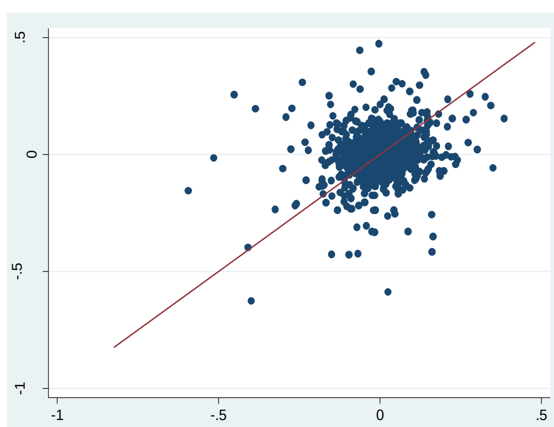
We estimated several alternative models via OLS and found that, using a simple criteria of minimum residual sum of squares, a model that included 14 factors in addition to the Fama-French factors gave the best fit.[15] The additional factors are in the category of patrimonial value is price over the total assets of the company; fixed assets over total assets measuring the fixed immobilized share of the value of the company; current liquidity measured as the ratio of current assets and current liabilities reflecting the capacity to pay of the company in the short term; and the relationship between debt and equity. Table 6 presents the estimation of a model that includes all factors and is the model that satisfies the criteria of best model in the sense that it has the lowest residual sum of squares under OLS estimation. The first column is the panel estimation with firm and year fixed effects. In this case, the beta is 0.612, and the constant is statistically zero. The estimated real return on equity in this model is 8.87%. The second column is the GMM estimation with firm and year fixed effects and instrumentation for lagged dependent variable and heteroskedasticity and autocorrelation (hac) robust standard errors. Beta increases to 0.612 and the constant is statistically zero. The estimated real return on equity in this model increases to 12.56%. Figure 4 shows the predicted versus actual real returns and the security and the GMM case is the best fit overall. The GMM model renders a higher beta and a higher estimated real cost of equity when all factors are included.

**Table 6.** Best Model via Residual Sum of Squares Firm and Year F.E.

| | **Panel premium_co** | **GMM premium_co** |
|---|---|---|
| **premium_mkt** | 0.612*** | 0.762*** |
| **book_to_mkt** | -0.0484* | 0.000524 |
| **Mkt_Value** | 0.00649 | 0.0136 |
| **P_Profit** | -0.00000451 | -0.00000433 |
| **Price_TotAssets** | 0.00277 | 0.00258 |
| **Price_Sales** | 0.0136*** | 0.00312 |
| **Ev_Ebitda** | -0.000466 | -0.000302 |
| **vix** | -0.000509 | 0.00418 |
| **Exch_rate** | 0.0169 | -0.129 |
| **Cred_GDP** | -0.000640 | -0.00562 |
| **Case_S** | 0.0000157 | 0.000179 |
| **Sharpe_Nasdaq** | -0.00732 | -0.00872 |
| **Fin_crisis** | 0.0240 | 0.0429 |
| **Fixed_Total** | -0.0000566 | -0.0000326 |
| **Leverage_Fin** | -0.00000530 | -0.00000109 |
| **Liquidity_curr** | -0.0122 | 0.00108 |
| **Debt_Equity** | 0.0194 | 0.0180 |
| **L.premium_co** | | 0.813 |
| **constant** | -.0147 | 0.052 |
| **Estimated return on equity in real terms** | 8.87% | 12.56% |
| **Sample size** | 885 | 841 |




**a.** Panel




**b.** GMM

**Figure 4.** Predicted versus Actual Real Returns

[15] Results of the several alternative models are available upon request. The residual sum of squares is one possible criteria, other could be in the related class of information criteria such as the AIC or BIC.

### 5.6 Nonlinearities

**Table 7.** Nonlinear Terms

| | (1) premium_co | (2) premium_co |
|---|---|---|
| **premium_mkt** | 0.438*** | 0.415*** |
| **premium_mkt2** | -0.506 | -0.377 |
| **premium_mkt3** | 6.961* | 7.798** |
| **book_to_mkt** | | -0.0385* |
| **Mkt_Value** | | 0.0207 |
| **constant** | 0.0158* | -0.430 |
| **Estimated return on equity in real terms** | 4.12% | 3.84% |
| **Sample size** | N=1495 | N=1345 |
| **adj. $R^2$** | 0.172 | 0.213 |

The case of nonlinear effects refers to the appending all models discussed above to include quadratic and cubic effects of the market premium as stated in expression (8). In particular, in the case where $\alpha_3 > 0$ it implies that the firm premium increases (decreases) as the positive (negative) skewness of the market becomes larger, i.e. preference for skewness. Table 7 shows models from the traditional CAPM (first column) and the Fama-French factors (second column) with firm and year fixed effects, and including the second and third order terms of equation (8). In the traditional CAPM, column (1), the beta is 0.438 and the constant is statistically different than zero thus rejecting the basic CAPM. The second order term is not statistically significant but the third order term is statistically significant and positive showing that investors show preference for skewness, i.e. the firm premium increases as the positive skewness of the market becomes larger. The estimated real return on equity in this model is 4.12% per year. In the CAPM with the Fama-French factors in column (2), the beta declines only slightly to 0.415, but the constant is statistically zero. The second order term is not statistically significant but the third order term is statistically significant and positive showing that investors show preference for skewness. The estimated real return on equity in this model declines to 3.84% per year.

We estimated the basic multiples model with firm and year fixed effects and including the second and third order effects. Beta declines 0.262, well below the 0.6 range and the estimated return on equity is 10.08% per year in real terms and the restriction that the constant is zero is satisfied in this case. The second order term is not statistically significant but the third order term is statistically significant and positive showing that investors show preference for skewness. Next, restricting the set of factors to Book-to Market; Price/sales; EV/EBITDA; and financial leverage; with firm fixed and year effects and adding the second and third order terms we obtain beta of 0.346 and estimated return of 9.70% per year in real terms. In this case, note that the restriction that the constant is zero is satisfied. The second order term is not statistically significant but the third order term is statistically significant and positive showing that investors show preference for skewness.

We also replicate the models with conditional heteroskedasticity, GMM and best models including the second and third order effects of equation (10). First, the models from the traditional CAPM, the Fama-French factors and the multiples with firm and year fixed effects and the higher order terms. In the traditional CAPM, Beta is much lower, 0.340, and the constant is statistically different than zero thus rejecting the basic CAPM. The arch and garch components of equation (6b) are significant. The arch in mean term in the main equation (6a) is not significant. The estimated real return on equity in this model is 7.41% per year. In the CAPM with the Fama-French factors, Beta declines further to 0.325, but the constant is statistically zero and arch and garch components of equation (6b) are significant. The estimated real return on equity in this model increases to 8.61% per year. In the CAPM with the Fama-French factors and the additional factors, beta remains at 0.323 and the constant is statistically zero. The arch and garch components are significant and arch in mean term is not significant. The estimated real return on equity in this model increases even further to 15.22% per year. The second order term is not statistically significant in all cases but the third order term is statistically significant and positive showing that investors show preference for skewness.

We estimated models from the traditional CAPM, the Fama-French factors and the additional multiples with firm and year fixed effects, but with the assumption that the error have a student's t distribution, i.e. slight fatter tails. In addition the higher order terms are included. In the traditional CAPM, beta is 0.378 and the constant is statistically different than zero thus rejecting the basic CAPM. The estimated real return on equity in this model is 4.59% per year, lower than the 7.41% per year under normality. In the CAPM with the Fama-French factors, the beta declines only slightly to 0.334, but the constant is statistically zero. The estimated real return on equity in this model decreases to 3.56% well below the 8.61% per year under normality. In the CAPM with the Fama-French factors and the additional factors, the beta declines further to 0.253 and the constant is statistically zero. The estimated real return on equity in this model increases even further to 10.84%, but below the 15.22% per year under normality. The arch and garch components are significant and arch in mean term is not significant in all cases above. The second order term is not statistically significant in all columns. The third order term is statistically significant only in two cases. Thus, it shows that investors have preference for skewness only if more factors are included when conditional heteroskedasticity is accounted for.

Overall, conditional heteroskedasticity effects are significant. Under student's *t* distribution, the estimated betas are slightly lower and the estimated real returns are lower than under normality. The second order term is not statistically significant but the third order term is statistically significant and positive showing that investors show preference for skewness, i.e. the firm premium increases as the positive skewness of the market becomes larger.

Next, we estimate models via generalized method of moments with instrumentation for lagged dependent variable and hetero-skedasticity and autocorrelation (hac) standard errors [(equation (7)] and include the higher order terms. Models from the traditional CAPM, the Fama-French factors and the multiples with firm and year fixed effects, but with GMM robust to heteroskedasticity and autocorrelation (hac). The traditional CAPM shows beta to be 0.407 and statically significant, while the constant is not statistically significant. The estimated real return on equity in this model is 5.43% . In the CAPM with the Fama-French factors, the beta declines 0.383, and the constant is statistically zero. The estimated real return on equity in this model is 6.25%. In the CAPM with the Fama-French factors and the additional factors, the beta declines to 0.353 and the constant is statistically zero. The estimated real return on equity in this model increases to 12.62%. More importantly, the second and third order terms are mostly not significant. Only with the Fama-French factors, the preference for s6kewness is significant.

**Table 8.** Nonlinear Terms Best Model via Residual Sum of Squares Firm and Year F.E.

| | **Panel premium_co** | **GMM premium_co** |
|---|---|---|
| **premium_mkt** | 0.261** | 0.344 |
| **premium_mkt2** | 0.951 | 0.551 |
| **premium_mkt3** | 18.33** | 18.60** |
| **book_to_mkt** | -0.0438** | -0.00852 |
| **Mkt_Value** | 0.00726 | 0.0156 |
| **P_Profit** | -0.00000414 | -0.00000369 |
| **Price_TotAssets** | 0.00326 | 0.00341 |
| **Price_Sales** | 0.0138*** | 0.00557 |
| **Ev_Ebitda** | -0.000467 | -0.000346 |
| **vix** | 0.000880 | 0.00467 |
| **Exch_rate** | -0.0918* | -0.214 |
| **Cred_GDP** | -0.00766 | -0.0131 |
| **Case_S** | 0.00117 | 0.00129 |
| **Sharpe_Nasdaq** | -0.000707 | 0.00000902 |
| **Fin_crisis** | 0.164* | 0.190* |
| **Fixed_Total** | -0.0000523 | -0.0000178 |
| **Leverage_Fin** | -0.00000133 | -0.00000164 |
| **Liquidity_curr** | -0.0140 | -0.00405 |
| **Debt_Equity** | 0.0190 | 0.0177 |
| **L.premium_co** | | 0.580 |
| **constant** | 0.00112 | 0.193 |
| **Estimated return on equity in real terms** | 9.26% | 13.28% |
| **Sample size** | 885 | 841 |

Table 8 presents the estimation of a model that includes all factors and is the model that satisfies the criteria of best model in the sense that it has the lowest residual sum of squares under OLS estimation as in Table 6. In addition the higher order terms are included. The first column is the panel estimation with firm and year fixed effects. In this case, the beta is 0.261 well below the beta of 0.612 in Table 6, and the constant is statistically zero. The estimated real return on equity in this model is 9.26% close to the number in Table 6. The second column is the GMM estimation with firm and year fixed effects and instrumentation for lagged dependent variable and heteroskedasticity and autocorrelation (hac) standard errors. Beta increases to 0.344 but it is not statistically significant. The constant is statistically zero. The estimated real return on equity in this model increases to 13.28% . In all columns the second order term is not statistically significant but the third order term is statistically significant showing that investors show preference for skewness. Figure 5 shows the predicted versus actual real returns and the security market space and the GMM case is a better fit overall.

In summary, the inclusion of nonlinear terms shows that in all cases mostly the second order term is not statistically significant. The third order term is mostly significant and positive indicating preference for skewness by investors. The exception is the estimation via GMM, where both the second and third order effects are mostly statistically negligible.

## 6. Comparison of Models for Beta and Predicted Real Equity

Figures 6-7 show comparisons of betas and predicted real equity returns from all models estimated as well as tables presenting statistics across models. Figure 6 shows betas centered at the point estimate plus and minus one standard deviation of the sample estimate. The first group of models without arch effects shows precise estimates of beta in the 0.6 range. Including arch effects provides precise beta estimates but in a range slightly lower, about 0.5. Some GMM models have the least precise estimates of beta and including the higher order terms decreases the range of beta significantly to the 0.3 range. The most precise estimates of beta are in the CAPM classic estimation with firm and year fixed effects accounting for Arch effects and with the student's *t* distribution to account for fatter tails. However, in this model the CAPM is rejected and we noted that this model is not a good fit for the data. The most imprecise estimates of beta are in the GMM cases of the classic CAPM. The four best models under residual sum of squares criteria with and without higher order terms have slightly less precise estimates of beta, however the fit of those models is significantly better in the back tests.

The first table denoted a. gives the overall average beta across the estimated models to be 0.468, the median is 0.451 and the standard deviation is 0.138. The mean plus one standard deviation is 0.607 and minus one standard deviation is 0.330. The second table denoted b. gives the statistics for the four best models under residual sum of squares criteria. The average beta across the best estimated models is slightly higher at 0.495, the median is 0.478 and the standard deviation is 0.233 indicating the lower precision mentioned above. The mean plus one standard deviation is 0.728 and minus one standard deviation is 0.262. We can conclude that for this sector in Brazil in this sample period, the beta is well under unity and in the range of 0.262-0.728.

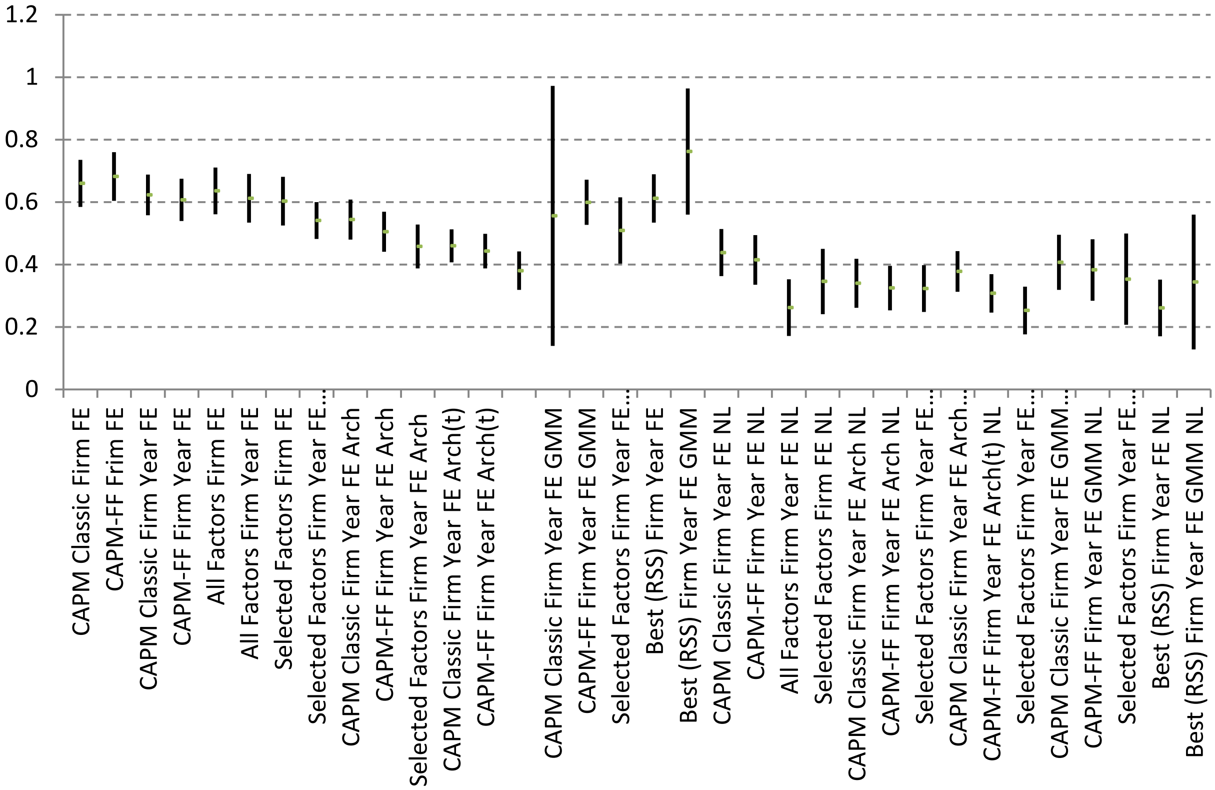



**Figure 6.** Betas Estimated with Alternative Econometric Models

**Mean, Median and Standard Deviation of Betas of All Alternative Models (Best Models in parentheses)**

| | Mean Beta | Median Beta | Standard Deviation | Mean, -1 StD | Mean, +1 StD |
|---|---|---|---|---|---|
| **Betas** | 0.468<br>(0.495) | 0.451<br>(0.478) | 0.138<br>(0.233) | 0.330<br>(0.262) | 0.607<br>(0.728) |

**Note:** In parentheses are corresponding values of Betas of best models

Figure 7 at the end shows the predicted real returns on equity per year for each model. The first four models of the classic and the CAPM with Fama-French factors have the lowest predicted real returns in all methodologies, below 4%. The lowest predicted real equity return is the CAPM with Fama-French factors and only firm fixed effects. The selected factors models have the highest predicted returns in the 14% range. The highest predicted return is the selected factors with firm and year fixed effects, arch and nonlinear terms. The four best models under residual sum of squares criteria with and without higher order terms have predicted returns in the range of 8.9% to 13.3%. The first table denoted a. gives the overall average predicted real return on equity across the models as 7.971%, the median is 8.010% and the standard deviation is 3.481%. The range of plus-minus one standard deviation is wide and gives a predicted real cost of equity as low as 4.490% to as high as 11.452%. The second table denoted b. gives the average predicted real return on equity across the four best models of Tables 6 and 8. The average across those models is 10.993%, the median is 10.910% and the standard deviation is 2.251%. The range of plus-minus one standard deviation gives a plausible predicted real cost of equity for Brazil in this sector and sample period of 8.742% to 13.243%.

## 7. Summary and Conclusions

We have provided an extensive set of alternative models for the estimation of the real cost of equity for a sample of utilities firms in Brazil in the last 6 years. Basically, we propose an extensive set of alternative multifactor models using multiples, risk factors and potential U.S. factors for the estimation of the real cost of equity for utilities firms in Brazil, which represent an important part of infrastructure, key to an emerging market. The main results are that the traditional CAPM and Fama-French three factors model are a poor fit and give the low betas and low estimated real return on equity. Additional factors improve the fit of the models, the estimated betas increase relative to the traditional CAPM and Fama-French, while the estimated real cost of equity also increase. Accounting for conditional heteroskedasticity effects shows that autocorrelation of variances is more important than news effects and under student's *t* distribution, the estimated betas are slightly lower and the estimated real returns are lower than under normality. The GMM models give a higher beta and a higher estimated real cost of equity when all factors are included, but the estimated betas are less precise in some cases. The inclusion of higher order terms shows that in almost all cases the second order term is not statistically significant. The third order term is mostly significant and positive indicating preference for skewness by investors. The exception is the estimation via GMM, where both the second and third order effects are mostly statistically negligible. The higher order terms impact little on the predicted real cost of equity relative to the cases where they are not included.

Our average estimates of the betas and the real cost of equity across all estimated models show that beta for this sector is well under unity and in the range of 0.33-0.61; and predicted real cost of equity in the range of 4.5% to 11.5%. However, several of those models are rejected and/or give a poor fit.

When we focus on the four best models under the criteria of least residual sum of squares, our average estimates of the betas show that it is well under unity, but in a slightly wider range of 0.26-0.73. This is expected since utilities are known to have stable cash flows and dividends. In this best case scenario, the predicted real cost of equity for Brazil in this sector and sample period is in the range of 8.7% to 13.2% averaging approximately 11% so that, from a public policy perspective, values within this range should be deemed plausible.

There are several potential avenues for future research. Most important is further work on the estimation of the real cost of debt and the weighted average cost of capital is a fruitful avenue from a public policy perspective.

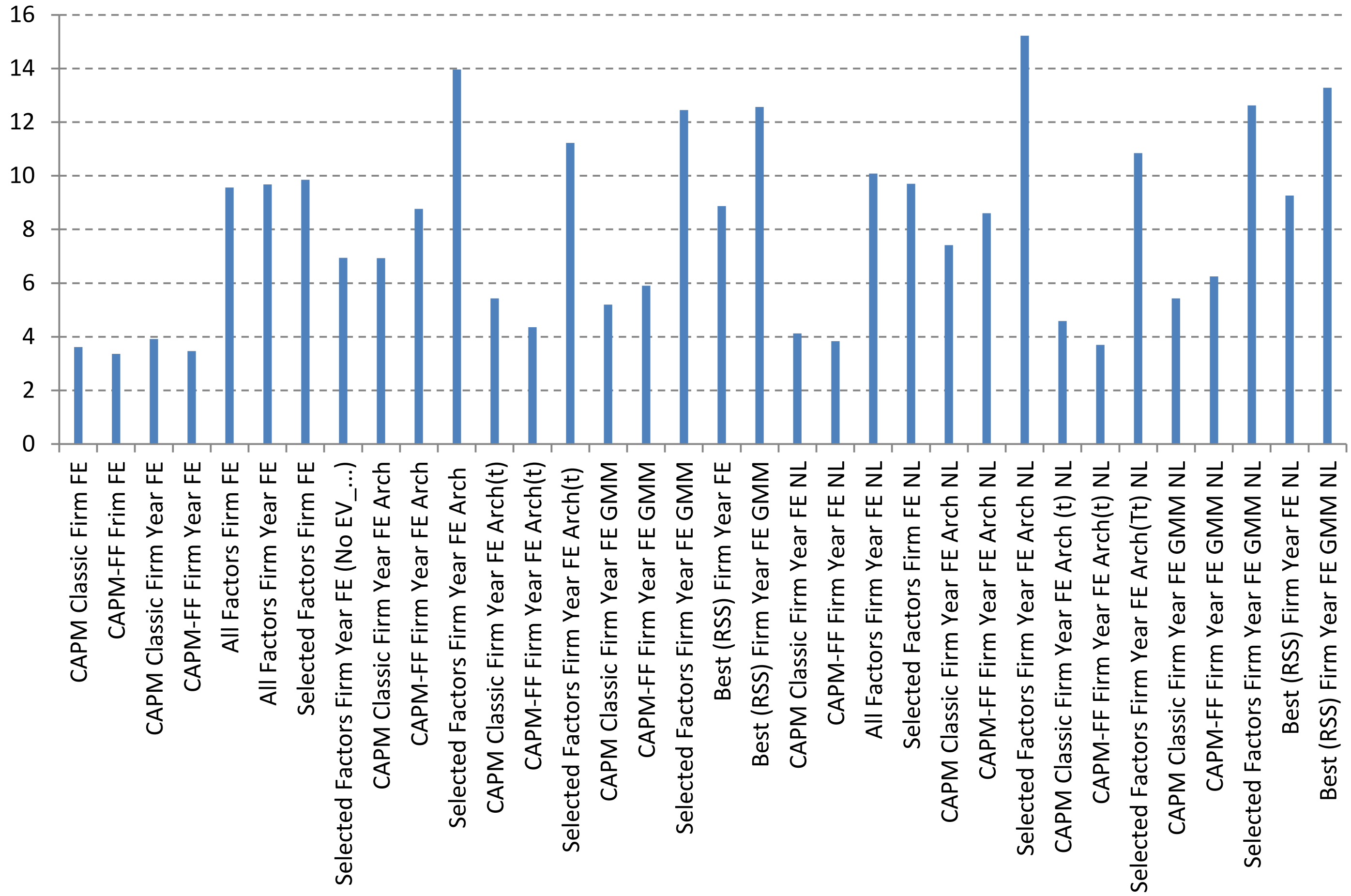



**Figure 7.** Predicted Real Equity Returns with Alternative Econometric Models

**Mean, Median and Standard Deviation of Predicted Real Equity Returns of All Alternative Models (% per year)**

| | Mean Real Return | Median Real Return | Standard Deviation | Mean -1 StD | Mean +1 StD |
|---|---|---|---|---|---|
| **Predicted Real Equity Returns** | 7.971% (10.99%) | 8.010% (10.91%) | 3.481% (2.251%) | 4.490% (8.742%) | 11.452% (13.24%) |

**Note:** In parentheses are corresponding Predicted Real Equity Returns of Best Models.

**Acknowledgments:** We thank the useful comments and suggestions of two anonymous referees and the editor for this Journal, the comments of Fernando Fernandes Neto; and Raphael Lolis and Bruno Hung for able research assistance. Any remaining errors are our own.

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