Intertemporal budget policies in an endogenous growth model with nominal assets

Marcelo Bianconi *

Department of Economics, Tufts University, Braker Hall, Medford, MA, 02155, USA

Abstract

Intertemporal budget policies are assessed in an endogenous growth model with nominal assets. The paper provides relative rankings of policies and policy instruments in terms of the tax liabilities of the private sector necessary to guarantee intertemporal government budget solvency and in terms of the welfare of the representative agent. The role of nominal assets is shown to be of relative importance. © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Growth; Government budget; Taxation; Expenditure

1. Introduction

When the problem of fiscal responsibility emerges in an economy, the role of fiscal and monetary policy instruments in relation to government budget deficits and debt is enhanced. This paper addresses the effects of alternative intertemporal budget policies based on fiscal and monetary instruments in an endogenous growth model driven by adjustment costs in investment. There is government debt and deficit in the model, and the comparative statics of fiscal and monetary policy on the long-term intertemporal governmental budget constraint are fully assessed. In particular, the author examines alternative policies regarding the long-term tax liability of the private sector that guarantee intertemporal solvency and the potential capital levies induced by nominal price effects.

One of the features of the model used in this paper is the introduction of adjustment costs to new investment, of the type analyzed by Hayashi (1982), in the class of AK

———
* Tel.: 617-627-2677; Fax: 617-627-3917.

 E-mail address: mbiancon@emerald.tufts.edu (M. Bianconi).

endogenous growth models as in Rebelo (1991). The introduction of adjustment costs had been only outlined in Barro and Sala-i-Martin (1992). Turnovsky (1996a,b) has taken the task of fully solving for an equilibrium path, most importantly showing the conditions under which an equilibrium exists. The current model builds on this literature by considering the problem of the intertemporal solvency of the government in the presence of government debt when there are three policy instruments available: capital income taxes, government spending, and the inflation tax. The marginal effects of these policy instruments operate through the cost of capital, say a q channel. Money is introduced directly in the utility function, as in Sidrausky (1967), under the usual rationale that it provides liquidity services.[1]

Two core budget policies are considered here. One is inspired by the contents of the Maastricht agreement of the European Community, in which countries joining the European Monetary Union (EMU) should obey fiscal rules consistent with a deficit to gross domestic product ratio of three percentage points or less, and ratios of public debt to gross domestic product of 60 percentage points or less. Along the balanced growth path in the current model, this policy implies a constant ratio of lump sum taxes to output subject to intertemporal solvency. The other budget policy is inspired by the United States Balanced Budget Amendment rule, such that the budget deficit is eliminated. A specific policy is designed that balances the budget deficit along the endogenous growth path. Among other things, a numerical simulation allows evaluation of the welfare costs and benefits of the balanced budget policy and its impact on the tax liabilities of the private sector. In general, welfare is inversely related to the capital income tax, government spending, and the inflation tax, whereas the tax liability is inversely related to the capital income tax and the inflation tax and is positively related to government spending.

There are three main results that emerge from this analysis that are worth noting. First, the introduction of money and inflation allows for examination of the role of the price level and the rate of inflation on the real tax liabilities of the private sector and on welfare, say the role of nominal assets in a fiscal policy framework. This effect turns out to be important and nontrivial. By ignoring it, one may leave aside variations in the tax liability of the private sector of the order of between 34 to 16.6 percentage points and on the stock of welfare of the private agent between 11.7 and −1.9 percentage points. Moreover, the theoretical possibility of dynamic "scoring" is shown to depend critically on the presence of nominal assets.[2] As a result, in the constant ratio of lump sum taxes to output policy, a necessary, but not sufficient, condition for dynamic scoring is that the consumer rate of time preference be strictly greater than the rate of growth of money. Alternatively, in the balanced budget policy it is sufficient and necessary that the inflation tax (price level) effect is present for dynamic scoring.

Second, introducing an arbitrary condition on the present discounted value of the tax liability for long-term intertemporal balance and adjusting the policy instruments to meet this condition imply welfare effects ranging from approximately a 2.1-percentage-point welfare gain when the capital income tax is used and approximately a 13.0-percentage-point welfare gain when the rate of growth of money is used versus approximately a 45-percentage-point welfare loss when government spending is used. Third,

relative rankings of the alternative policies and policy instruments are shown. The balanced budget policy is the one that consistently deteriorates the lump sum tax liabilities of the private sector. Also, marginal cuts in government spending provide welfare gains and reductions in the tax liability of the private sector that make it the most attractive policy instrument, more so in the constant ratio of lump sum taxes to output policy relative to the balanced budget policy.

The paper is organized as follows: section 2 presents the model and its solution together with the conditions for existence of an equilibrium growth path; section 3 presents comparative static results along the growth path; section 4 introduces the analytics of the intertemporal government budget constraint; section 5 introduces the alternative policies regarding the lump sum tax liability of the private sector, the design of a balanced budget policy, the long-term intertemporal balance issue, and a numerical assessment of the comparative statics, and welfare costs of the alternative policies. Section 6 concludes the paper.

2. Macroeconomic structure and general equilibrium

The model is a decentralized, one-sector endogenous growth model with three assets: physical capital, government bonds, and money. The building blocks are as follows.

2.1. Households

The representative household solves the intertemporal problem in Eq. (1):

$$\text{Max} \int_0^\infty U(c,m) \, (\exp - \delta t) dt \tag{1}$$

subject to the household budget constraint choosing $\{c, m, k, I, b\}$ [Eq. (1a)],

$$c + \dot{m} + \dot{b} + \Phi(I,k) = (l - \tau_k)r_k k + b(r_b - \pi) - \pi m - T, \tag{1a}$$

the capital accumulation rule [Eq. (1b)],

$$\dot{k} = I, \tag{1b}$$

and given initial holdings [Eq. (1c)],

$$k_o > 0, \qquad M_o > 0, B_o, \tag{1c}$$

where $m \equiv M/P$ is the stock of real money balances, $b \equiv B/P$ is the stock of real government bonds, P is the price level, c is private consumption, $\delta \geq 0$ is the consumer subjective rate of time preference, I is real investment, k is the stock of physical capital (assuming no rate of depreciation), r_k is the real interest earned on physical capital taxed at flat rate $0 \leq \tau_k < 1$, r_b is the nominal interest on government bonds, π is the rate of inflation, $(r_b - \pi)$ is the real interest earned on the stock of real government bonds, and T is a government real lump sum tax.

The representative household solves a standard problem in Eq. (1). The domestic household budget constraint is expressed in real flow terms and consists of after-tax physical capital income, interest income on bonds (assumed not to be taxed) minus

the inflation tax minus lump-sum taxes (right-hand-side), to be spent on gross consumption or gross additions to the stock of physical capital, money balances, or government bonds (left-hand-side).[3]

The functions $U(.\,,.)$ and $\Phi(.\,,.)$ are assumed to take the following specific forms [Eqs. (2a,b)]:

$$U(c,m) \equiv \log c + \gamma \log m, \quad \gamma < 0, \tag{2a}$$

and

$$\Phi(I,k) \equiv I[1 + (h/2)(I/k)], \quad h \geqslant 0. \tag{2b}$$

These are chosen for analytical tractability and are standard. Utility is unitary elastic with γ representing the degree of liquidity services provided by real money balances in total utility. It will be useful to consider a polar case where $\gamma \to 0$ as a proxy for no role of nominal assets.[4] The investment function is quadratic and belongs to the adjustment cost class of investment models as in Hayashi (1982), with h denoting the sensitivity of the investment function to the quadratic cost.

An interior solution for Eq. (1) with functional forms as in Eq. (2) is obtained by considering the set of first order conditions with λ and q' denoting the Lagrange multipliers associated to the household budget constraint and the investment accumulation equation, respectively. These are given by

$$c\,\lambda = 1, \tag{3a}$$

$$\dot{\lambda} = (\delta + \pi)\lambda - (\gamma/m), \tag{3b}$$

$$\lambda[(1 - \tau_k)r_k + (h/2)(I/k)^2] = -\dot{q}' + q'\delta, \tag{3c}$$

$$\lambda[1 + h(I/k)] = q', \tag{3d}$$

$$\lambda(r_b - \pi) = -\dot{\lambda} + \lambda\delta, \tag{3e}$$

together with the transversality conditions [Eq. (3f)]

$$\lim_{t\to\infty} \lambda m(\exp - \delta t) = 0, \ \lim_{t\to\infty} \lambda b(\exp - \delta t) = 0, \ \text{and} \ \lim_{t\to\infty} q'k(\exp - \delta t) = 0. \tag{3f}$$

Eqs. (3a and b) are marginal conditions for consumption and real money balances, Eq. (3d) relates the marginal utility of consumption to the marginal cost of investing, and Eqs. (3c and 3e) are arbitrage conditions equating rates of return across holdings of assets and consumption.

2.2. Firms

The representative firm is assumed to operate with a linear technology implying point-in-time constant returns to scale over a broad measure of capital, k. This is given by $f(k) = \alpha k$, for a constant $\alpha > 0$. The argmax of the profit function $f(k) - r_k k$ implies that Eq. (4) holds:

$$f'(k) = r_k = \alpha. \tag{4}$$

2.3. Government behavior and goods market equilibrium

The government can finance its expenditure activities of non–utility-enhancing real consumption, g, minus real lump sum taxes, T, plus real interest on outstanding debt,

$(r_b - \pi)b$, by issuing new money or government bonds, or by taxing capital income or real money holdings. The flow government budget constraint in real terms is given by:

$$\dot{m} + \dot{b} = g - T + (r_b - \pi)b - \tau_k r_k k - \pi m. \tag{5a}$$

Government monetary policy is a simple constant rate of growth of money type rule, which implies an evolution of the stock of real money balances given by Eq. (5b):

$$\dot{m} = (\sigma - \pi)m, \tag{5b}$$

where $\sigma \equiv \dot{M}/M$ is the nominal monetary growth rate.

Government consumption expenditure is assumed to be proportional to output, αk, with $g(t) = g^*\alpha k$ for a constant $g^* \geqslant 0$ chosen exogenously. The lump sum tax policy is also assumed to be related to output, but with some flexibility in terms of the exact proportion and its dependence on time. Hence, $T(t) = T^*(t)\alpha k$ for $T^*(t)$, free to be chosen exogenously by the government or endogenously to accommodate for an equilibrium.

The household budget constraint in Eq. (1a) plus the government budget constraint in Eq. (5a) give the goods market equilibrium denoted by

$$f(k) = \alpha k = c + I[1 + (h/2)(I/k)] + g. \tag{6}$$

The wealth of the representative individual at any point in time is the sum of the three assets: $k + m + b$.

1.4. The general equilibrium growth path

The first order condition for I in Eq. (3d) gives the essence of the q-growth model. Defining $q \equiv q'/\lambda$ to be the relative price of capital to the (unitary) price of bonds, one finds that

$$I/k = \dot{k}/k = (q - 1)/h \equiv \eta_k, \tag{7a}$$

which implies a growth path for the capital stock given by $k(t) = k_o[\exp\int_0^t \eta_k(s) \ ds]$, for a $k_o > 0$ given. Eq. (7a) is similar to the traditional Tobin (1969) equation describing Tobin's q.

The first-order conditions for consumption, Eq. (3a) and bonds and Eq. (3e) imply that

$$\dot{c}/c = -\dot{\lambda}/\lambda = [(r_b - \pi) - \delta] \equiv \eta_c, \tag{7b}$$

which implies time paths for the consumption and the marginal utility given by $c(t) = c(0) [\exp\int_0^t \eta_c(s) \ ds]$, and $\lambda(t) = \lambda(0)[\exp\int_0^t - \eta_c(s) \ ds]$ for $c(0)$ and $\lambda(0)$ to be determined endogenously.

At this point, we conjecture that the equilibrium is one where $q(t)$ is constant for $t > 0$. In this case, because g and k and I and k grow proportionally, c and k must grow proportionally to maintain the goods market equilibrium, because this is a closed economy. Hence, it must be the case that the growth rates are equalized: $\eta_k = \eta_c \equiv \eta$. This implies that

$$(q - 1)/h = [(r_b - \pi) - \delta]. \tag{7c}$$

Using the first-order conditions for c, k, and I, or Eqs. (3a, c, and d), respectively, the maximum profit condition in Eq. (4), and Eq. (7c), one obtains a quadratic equation for the conjectured constant q given by

$$q^2 + 2h\delta q - [1 + 2h\alpha(1 - \tau_k)] = 0, \tag{7d}$$

which has two real roots, one negative (ruled out), and another possibly greater than one given by

$$q_o = -h\delta + [1 + h^2 \delta^2 + 2h\alpha (1 - \tau_k)]^{1/2}. \tag{7e}$$

For this plausible root, $d\dot{q}/dq = \delta + q_o/h > 0$, which proves the conjecture that $q(t)$ is constant for $t > 0$. There are no transitional dynamics in the shadow price of capital such that it takes the appropriate initial jump at $t = 0$ to guarantee a stable equilibrium. Throughout the analysis, we assume that q_o is strictly greater than one, $q_o > 1$, such that it guarantees an equilibrium path with positive endogenous growth. This imposes a restriction on the parameters of the model of the form $\alpha (1 - \tau_k) > \delta$, which is satisfied for a set of plausible parameter values.[5]

The endogenous initial level of consumption and marginal utility are then determined from the goods market equilibrium, Eq. (6), as

$$c(0) = \alpha k_o \{1 + [(1 - q_o^2)/2h\alpha] - g^*\} \tag{8}$$

with $\lambda(0) = 1/c(0)$.

The determination of real money balances is as follows. Using the first-order conditions for m and c to substitute for the rate of inflation in the real money balances, Eq. (5b) gives

$$\dot{m} - (\sigma + \eta + \delta)m = -\gamma c. \tag{5b$'$}$$

Integrating Eq. (5b$'$), one obtains Eq. (5b$''$),

$$m(t) = [\exp(\sigma + \eta + \delta)t] \{m(0) + [\gamma c(0)/(\sigma + \delta)]([\exp - (\sigma + \delta)t] - 1)\}, \tag{5b$''$}$$

which, on application of the relevant transversality condition, implies

$$m(0) = [\gamma/(\sigma + \delta)] c(0). \tag{9}$$

Hence, the solution for the initial stock of real money balances implies that it grows at the same rate as consumption and the capital stock, η. Given an initial stock of nominal money balances, the price level, $P(0)$, jumps initially to guarantee an equilibrium growth path where all variables grow at the same rate with no transitional dynamics for real money balances. The solution for the rate of inflation follows as

$$\pi = \sigma - \eta, \tag{10}$$

the nominal interest rate on government bonds is

$$r_b = \delta + \sigma, \tag{11a}$$

and the real interest rate is from above,

$$r_b - \pi = \delta + \eta. \tag{11b}$$

The discounted value of instantaneous welfare in Eq. (1), denoted by Z, is given by

$$Z = [(1 + \gamma)/\delta] \log c(0) + [(1 + \gamma)/\delta^2]\eta + (\gamma/\delta) \log [\gamma/(\sigma + \delta)], \tag{12}$$

which represents a stock measure of welfare for the representative individual. The welfare changes will be computed as marginal changes in this stock, given marginal changes in the policy instruments.

The equilibrium growth path is then a solution for q_o, η, $c(0)$, $m(0)$, π, r_b, and Z, given policy parameters τ_k, g^*, and σ, where k, c, and m grow at the same rate. This is very much a classical monetary model with an endogenous growth path. The initial endogenous jumps in the marginal utility of consumption and the price level guarantee instantaneous equilibrium in the goods and money markets such that the economy adjusts to its equilibrium path without transitional dynamics.

The effect of adjustment costs in the endogenous growth framework, the q-growth effect, is shown by the following two expressions [Eqs. (13a&b):

$$\partial q_o/\partial h = [h\delta^2 + (1 - \tau_k)\alpha][1 + h^2\delta^2 + 2h(1 - \tau_k)\alpha]^{-1/2} - \delta \tag{13a}$$

and

$$\partial\eta/\partial h = (1/h^2)\{1 - [1 + h(1 - \tau_k)\alpha]/(q_o + h\delta)\}. \tag{13b}$$

The signs of both $\partial q_o/\partial h$ and $\partial\eta/\partial h$ can be positive or negative. The sign of $\partial\eta/\partial h$ is guaranteed to be negative if $q_o > \{1 + h[(1 - \tau_k)\alpha - \delta]\} > 1$. Under this condition, an increase in the slope of the marginal cost of investing an additional unit, $\partial h > 0$, has a negative impact on the growth rate of the economy.

3. Monetary and fiscal policy: Some preliminaries

The marginal effects of the policy instruments on the equilibrium growth path can be computed from the equilibrium conditions in Eqs. (7–12), above. A marginal change in the capital income tax rate gives Eqs. (14a&b):

$$\partial q_o/\partial\tau_k = -h\alpha[1 + h^2\delta^2 + 2h\alpha(1 - \tau_k)]^{-1/2} < 0 \tag{14a}$$

and

$$\partial\eta/\partial\tau_k = -\alpha[1 + h^2\delta^2 + 2h\alpha(1 - \tau_k)]^{-1/2} < 0. \tag{14b}$$

The capital income tax rate has the well-known contractionary effect on the price of capital and consequently on the growth rate of the economy. These imply that $\partial c(0)/\partial\tau_k > 0$, because agents substitute investment for consumption. There is an associated price level effect. The price level falls because given the constant path of nominal money balances, the induced increase in the real money demand must be accommodated to maintain the money market in equilibrium, hence $\partial m(0)/\partial\tau_k > 0$. The negative growth effect increases the rate of inflation, $\partial\pi/\partial\tau_k > 0$, but the nominal return on government bonds is unchanged, $\partial r_b/\partial\tau_k = 0$, such that its real return falls $\partial(r_b - \pi)/\partial\tau_k < 0$. The welfare effect of the change in the capital income tax, $\partial Z/\partial\tau_k$, is ambiguous because the consumption and growth effects in Eq. (12) go in opposite direction.[6]

The effects of a marginal change in the share of government spending, g^*, on the price of capital and the growth rate are trivial as may be seen from Eqs. (7a and e), $\partial q_o/\partial g^* = \partial \eta/\partial g^* = 0$. Private consumption falls through the usual crowding out effect, $\partial c(0)/\partial g^* < 0$, and the price level increases given the fall in the demand for real money balances, $\partial m(0)/\partial g^* < 0$. The effects on inflation and the nominal and consequently real return on government bonds are trivial, $\partial \pi/\partial g^* = \partial r_b/\partial g^* = \partial(r_b - \pi)/\partial g^* = 0$, whereas welfare is negatively related to g^* given the crowding out mechanism, $\partial Z/\partial g^* < 0$.

Finally, the effects of a marginal increase in the rate of growth of money on q_o, η, $c(0)$, and $(r_b - \pi)$ are all null, $\partial q_o/\partial \sigma = \partial \eta/\partial \sigma = \partial c(0)/\partial \sigma = \partial(r_b - \pi)/\partial \sigma = 0$. This is a classical monetary model in the tradition of Sidrausky (1967) in the sense that money has the usual inflation tax role, the price level increases to equilibrate the money market, $\partial m(0)/\partial \sigma < 0$, and the anticipated inflation effect implies that $\partial \pi/\partial \sigma = 1 > 0$ and $\partial r_b/\partial \sigma = 1 > 0$. The welfare effect is negative because of the liquidity services attribute of real balances, $\partial Z/\partial \sigma < 0$.

4. The intertemporal government budget constraint

Substituting all the equilibrium conditions on the intertemporal government budget constraint in Eq. (5a), one obtains

$$\dot{b} - (\delta + \eta)b = \alpha k_o d_o(\exp \eta t) - \alpha k_o T^*(t)(\exp \eta t), \qquad (15a)$$

where $d_o \equiv \{g^* - \tau_k - [\sigma\gamma/(\sigma + \delta)][c(0)/\alpha k_o]\}$ is defined as the deficit net of interest payments and transfers as a share of output, say some measure of the "primary" deficit. Note that the effective discount rate on government debt is the real interest rate, $\delta + \eta$. Integrating Eq. (15a) and applying the relevant transversality condition implies the stock constraint

$$\int_0^\infty T^*(t)(\exp - \delta t)dt = [b(0)/\alpha k_o] + (d_o/\delta), \qquad (15b)$$

which, given $b(0) = B_o/P(0)$ and d_o, imply that the path of the lump sum share, $T^*(t)$, must be endogenous to satisfy this constraint.[7] The stock of government debt at each point in time is then given by Eq. (15c):

$$b(t) = [\exp(\delta + \eta)t] \{\alpha k_o \int_t^\infty T^*(t)(\exp - \delta t)dt - (\alpha k_o d_o/\delta)\}, \qquad (15c)$$

which implies that $b(t)$ may grow at a rate that is different from η, depending ultimately on the time path of lump sum taxes chosen by the policy maker.

From Eq. (15b), given that $T(t) = T^*(t)\alpha k$, define

$$V(T) \equiv \int_0^\infty T(t)[\exp -(\eta + \delta)t]dt = b(0) + (\alpha k_o d_o/\delta), \qquad (15b')$$

where $V(T)$ is the present discounted value of real lump sum taxes required to satisfy the intertemporal government budget constraint. This quantity can be thought of as a measure of the tax liability of the private sector necessary to balance the intertemporal government budget constraint for a given $b(0) > 0$ and $d_o > 0$.[8]
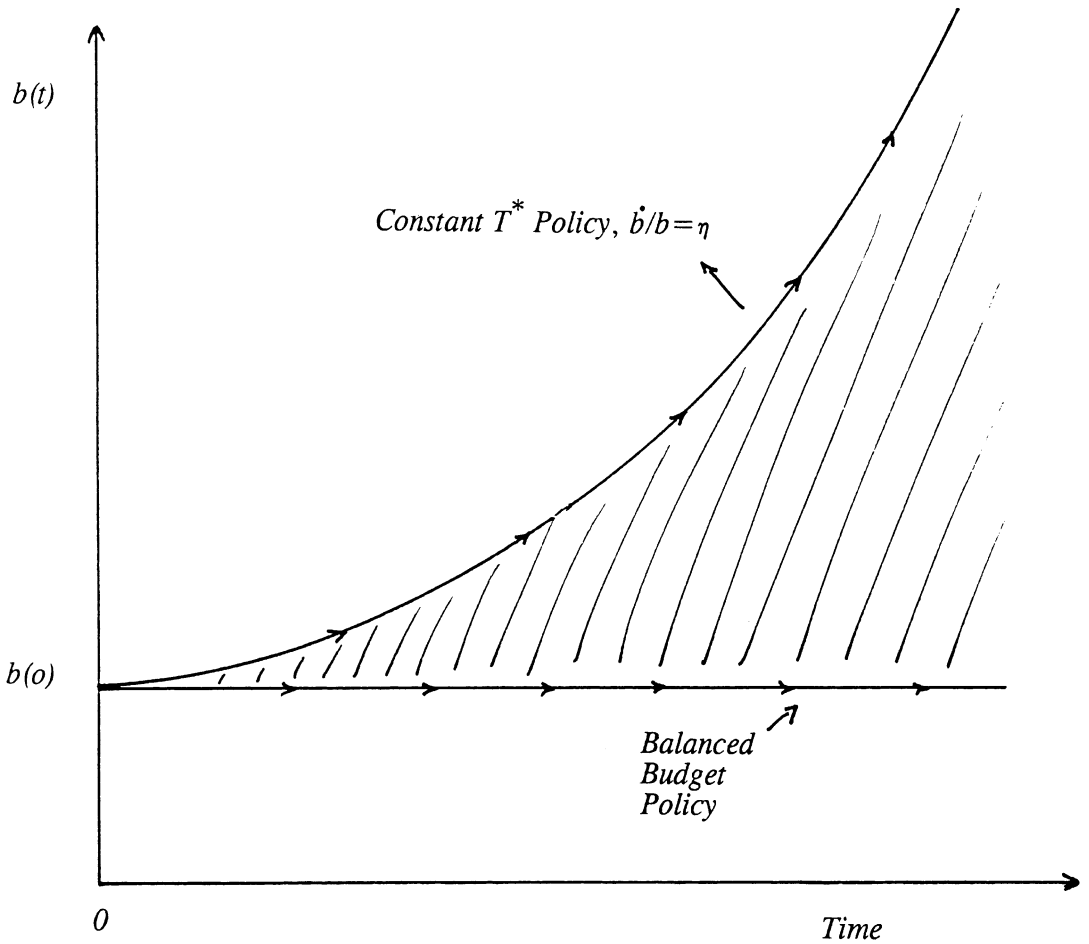
Fig. 1. Time path of government debt.

5. Alternative tax, debt, and deficit policies

5.1. A constant T* policy

The first policy choice for the path of lump sum taxes in Eq. (15b′) is to assume that $T*$ is constant. This budget policy is consistent with the Maastricht agreement of the European Community: the lump sum tax rule implies a constant deficit-to-output ratio and a constant public-debt-to-output ratio. In this case, $b(t)$ grows at the constant rate, η (see, e.g., Fig. 1), along with k, c, and m, and the present discounted value of real lump sum taxes, or the tax liability, is given by Eq. (15b′) itself, or Eq. (15b″):

$$V(T)|_{T*\text{constant}} = b(0) + (\alpha k_o d_o/\delta). \tag{15b″}$$

Marginal changes in monetary and fiscal policy will have various effects on the tax

bases, on the real value of the initial stock of government debt, and on the growth rate that will ultimately affect the tax liability of the private sector. These effects can be generally interpreted as equivalent to the determination of the slope of the monetary Laffer curve in a general equilibrium approach.

For example, the marginal effect of a change in the capital income tax rate is given by

$$\frac{\partial V(T)}{\partial \tau_{k|T*\text{constant}}} = -(\alpha k_o/\delta)\{1 + (\delta - \sigma)[q_o \gamma/h\alpha(\sigma + \delta)](\partial q_o/\partial \tau_k)\}. \tag{16a}$$

Eq. (16a) is seen to depend on the effect of the capital income tax on the cost of capital, $\partial q_o/\partial \tau_k < 0$, say the q-channel, and more importantly on the sign of $(\delta - \sigma)$, the discrepancy between the rate of time preference and the rate of growth of money. If $(\delta - \sigma) \leq 0$, then $\partial V(T)/\partial \tau_{k|T*\text{constant}} < 0$ unambiguously, and there is an increase in the long-term liability of the household if capital income taxes are cut. If $(\delta - \sigma) > 0$, then the sign of $\partial V(T)/\partial \tau_{k|T*\text{constant}}$ is ambiguous. It is possible in this case that $\partial V(T)/\partial \tau_{k|T*\text{constant}} > 0$ and a decrease in capital income taxes could in fact reduce the long-term liability of the household, some sort of dynamic "scoring" result, see, for example, Bruce and Turnovsky (1995). This result derives from the possibility of the inflation tax and the induced increase in the price level effect to dominate, because the sign of $(\delta - \sigma)$ balances these effects against the pressure of lower capital income taxes on the intertemporal imbalance. Thus the presence of nominal assets is critical for the possibility of dynamic scoring. The analogy of Laffer-style effects in this case is clear. A decrease in the tax rate on capital increases the price level leading to two opposing effects: (i) a raise in the price level leads real money balances to decrease, thus decreasing the revenues from money creation; (ii) the higher price level decreases the real value of initial debt, say a capital levy. Roughly, if (ii) dominates (i), we obtain dynamic scoring.

A marginal change in the share of government spending in total output is given by

$$\frac{\partial V(T)}{\partial g*|_{T*\text{constant}}} = \left(\frac{\alpha k_o}{\delta}\right)\left\{1 - (\delta - \sigma)\left[\frac{\gamma}{(\sigma + \delta)}\right]\right\}. \tag{16b}$$

Also, Eq. (16b) is seen to depend critically on the sign of $(\delta - \sigma)$. If $(\delta - \sigma) \leq 0$, then $\partial V(T)/\partial g*|_{T*\text{constant}} > 0$ unambiguously, and there is an increase in the long-term liability of the household if the government spending share increases. If $(\delta - \sigma) > 0$, then the sign of $\partial V(T)/\partial g*|_{T*\text{constant}}$ is ambiguous. It is possible in this case that $\partial V(T)/\partial g*|_{T*\text{constant}} < 0$ and an increase in the government spending share could reduce the long-term liability of the household. Again, the inflation tax and price level effects may dominate, because the sign of $(\delta - \sigma)$ balances these against the pressure of higher spending on the intertemporal imbalance.

A marginal change in the rate of growth of money implies Eq. (16c):

$$\frac{\partial V(T)}{\partial \sigma|_{T*\text{constant}}} = -2\gamma c(0)/(\sigma + \delta)^2 < 0, \tag{16c}$$

which is unambiguously negative. A reduction in the inflation tax implies an unambigu-

ous increase in the long-term tax liability of the household; there is no scope for any dynamic interaction that will revert the loss of inflation tax revenues.

In these three cases, it is important to note that the price level effect is quite critical. In the polar case when $\gamma \to 0$, say money and nominal prices play no role, $\partial V(T)/\partial \tau_k|_{T^*\text{constant}} = -\partial V(T)/\partial g^*|_{T^*\text{constant}}$ are unambiguously negative and positive, respectively, and $\partial V(T)/\partial \sigma|_{T^*\text{constant}} = 0$ because the inflation tax role disappears. The possibility of dynamic scoring either with capital income taxes or with government spending is ruled out because the inflation tax and the nominal price channel of a revaluation of government assets is not present anymore. Thus, in this framework, the possibility of these scoring interactions depends critically on the nominal effects brought about by the presence of nominal assets.

5.2. A balanced budget policy

In the endogenous q-growth model with government debt presented here, the stock of government debt grows along the balanced growth path, with its particular rate of growth depending on the specific policy chosen regarding the path of lump sum taxes. This implies that there is an ongoing budget deficit in the economy along the endogenous growth path, even though at the infinite horizon, intertemporal solvency is guaranteed. However, the United States Balanced Budget Amendment rule asks for a policy of zero budget deficit. In this section, a policy that eliminates the budget deficit along the endogenous growth path is considered.

Fig. 1 illustrates the main objective of the balanced budget policy. For example, the time path of government debt grows at the constant rate, η, in the constant T^* policy, from an initial level of debt, $b(0)$. Suppose at time $t = 0$, the policy maker implements a policy of a balanced budget deficit, with the stock remaining constant at $b(0)$. A feasible policy must raise the resources to cover the shaded area in Fig. 1 and must satisfy the intertemporal solvency condition simultaneously. Formally, this policy must be consistent with

$$\dot{b}(t) = 0 \quad \text{with} \quad b(t) = b(0) \qquad \forall t \geqslant 0. \tag{17a}$$

The unique policy that attains this objective is the following:

  (i)   choose one of the policy instruments, $\{\tau_k, g^*, \sigma\}$, to set $d_o(\tau_k, g^*, \sigma) \equiv 0$;

  (ii) set $T^*(t) = T^*_o (\exp - \eta t)$;

  (iii) set $T_o = (\eta + \delta) b(0) \qquad \forall t \geqslant 0.$ $\tag{17b}$

Part (i) eliminates the "primary" deficit by choice of one of the three policy instruments, whereas the other two instruments remain free. Part (ii) implies that $T^*(t)$ declines monotonically at the rate η, whereas part (iii) gives the constant initial lump sum tax consistent with intertemporal solvency. This guarantees that the implied budget deficit, Eq. (17c), holds:

$$\dot{b}(t) = \{\alpha k_o T^*_o (\eta - \eta)[\exp(\eta - \eta)t]/(\delta + \eta)\} + [\alpha k_o d_o \eta(\exp \eta t)/\delta] = 0$$
$$\forall t \geqslant 0. \tag{17c}$$

The present discounted value of real lump sum taxes on implementation of this balanced budget (BB) policy by Eqs. (15b′) and (17b) is given by

$$V(T)|_{BB} = b(0), \tag{17d}$$

which is just the initial outstanding real debt.

We can compute the marginal effect of changing the alternative policy instruments to balance the primary deficit in the neighborhood of the equilibrium set out by the policy in Eq. (17b). Hence, a marginal change in the capital income tax to balance the primary deficit to implement this policy has an effect on the present discounted value of lump sum taxes in Eq. (17d) given by

$$\partial V(T)/\partial \tau_{k|BB} = -(\partial q_o/\partial \tau_k)\left[\frac{q_o k_o \gamma}{(\sigma + \delta)h}\right] > 0. \tag{17e}$$

Again, this is seen to depend on the effect of the capital income tax on the cost of capital, $\partial q_o/\partial \tau_k < 0$, say the q-channel. Note that $\partial V(T)/\partial \tau_{k|BB} > 0$ and a decrease (increase) in capital income taxes to implement the balanced budget policy would in fact reduce (increase) the long-term liability of the household. There is scope for dynamic scoring because of the inflation tax (price level) effect on the level of initial debt. Note that this is an effect in the neighborhood of the balanced budget policy such that the only effect that counts is the change in real debt given by the change in the price level triggered by the change in the capital income tax.

The marginal effect of a change in the share of government spending in total output to implement the balanced budget policy is given by

$$\partial V(T)/\partial g^*|_{BB} = -\gamma \alpha k_o/(\sigma + \delta) < 0, \tag{17f}$$

which is unambiguously negative. There is a decrease (increase) in the long-term liability of the household if the government spending share increases (decreases) to implement the balanced budget policy.

A marginal change in the rate of growth of money in this case implies that

$$\partial V(T)/\partial \sigma|_{BB} = -\gamma c(0)/(\sigma + \delta)^2 < 0. \tag{17g}$$

When σ is set to implement the balanced budget policy, an inverse effect in the long-term liability of the household obtains. This is the direct effect of the inflation tax and price level on the initial stock of government debt.

The price level effect is again of critical importance. In the polar case when $\gamma \to 0$, say nominal assets play no role, there will be no effect on the present value of the tax liability, thus no possibility of dynamic scoring. The inflation tax (price level effect) is the only channel, and $\gamma \to 0$ renders it null. Note, however, that the comparative statics in Eqs. (17e, f, and g) are all in the neighborhood of the balanced budget equilibrium, and thus are relative to this specific balanced budget policy. We use numerical simulations below to examine a full comparison across the two policies.

5.3. The long-term intertemporal balance

The stock constraint in Eq. (15b′) and the policies studied in Eqs. (15b″)–(17a) highlight one issue that is important. The intertemporal solvency of the government

is shown to depend on the present discounted value of the tax liability. One measure of long-term intertemporal balance for the government is to allow one of the policy instruments to adjust endogenously and satisfy the constraint that $V(T)$ be less than or equal to zero: $V(T) \leq 0$.[9]

In the constant T^* case of Eq. (15b″), the long-term intertemporal balance constraint translates into the condition

$$b(0) + \left(\frac{\alpha k_o d_o}{\delta} \right) \leq 0, \tag{18a}$$

and one of the three policy instruments, $\{\tau_k, g^*, \sigma\}$, is chosen endogenously to satisfy Eq. (18a). However, in the balanced budget policy, BB, the problem is somewhat more subtle. In part (iii) of Eq. (17b), the lump sum tax is imposed and paid at the instant of implementation, at $t = 0$. Hence $V(T)|_{BB} \leq 0$ if and only if $b(0) \leq 0$, that is, the government cannot start with an initial debt independently of the policy instruments $\{\tau_k, g^*, \sigma\}$. In the constant T^* case, one of the government parameters can be chosen endogenously to satisfy the constraint Eq. (18a). Any of $\{\tau_k, g^*, \sigma\}$ are going to be chosen, obviously to satisfy Eq. (18a) with equality, giving any choice of

$$\tau_k = \left(\frac{\delta b(0)}{\alpha k_o} \right) + g^* \left[1 + \left(\frac{\delta \sigma}{\sigma + \delta} \right) \right] - \left(\frac{\delta \sigma}{\sigma + \delta} \right) \left\{ 1 + \left[\frac{(1 - q_o^2)}{2h\alpha} \right] \right\}, \tag{18b}$$

$$g^* = \frac{\tau_k(\sigma + \delta)}{(\sigma + \delta + \sigma\delta)} + \left[\frac{\sigma\delta}{(\sigma + \delta + \sigma\delta)} \right] \left\{ 1 + \left[\frac{(1 - q_o^2)}{2h\alpha} \right] \right\} - \frac{\delta b(0)(\sigma + \delta)}{\alpha k_o(\sigma + \delta + \sigma\delta)}, \tag{18c}$$

or

$$\sigma = \frac{\delta \{ \delta b(0) + \alpha k_o(g^* - \tau_k) \}}{\{ \gamma c(0) - [\delta b(0) + \alpha k_o(g^* - \tau_k)] \}}, \tag{18d}$$

and the numerical simulations below can be used to assess the welfare effects of satisfying the constraints of Eqs. (18b–d).

5.4. Numerical simulations

To assess the impact of the alternative policies on the liabilities of the private sector that guarantee intertemporal solvency and to examine the welfare effects of the alternative policies, we resort to a simple numerical simulation of the model. The benchmark set of parameter values is given in the bottom of Table 1 and is a plausible one where there is positive endogenous growth. It implies an equilibrium 2% endogenous growth path, the consumption share is approximately 53%, the velocity of circulation is approximately 6, the initial stock of government debt is 50% of output, the initial primary surplus is 5.7% of output (as is roughly the current United States case), inflation is 2.125%, and the real return on government bonds is 6%. This parameterization implies lump sum tax credits of the order of 92% of output to guarantee long-term intertemporal solvency.[10]

Table 1 summarizes the effects of the alternative intertemporal budget policies and

Table 1
Tax liabilities and the welfare cost of budget policies

	Percent changes			
	$V(T)$[a]	$V(T)$	Z	$X\|_{LC}$
Constant T^* policy				
Benchmark set[b]	0	—	0	—
$\partial\tau_{k\|T^*\text{constant}} < 0$, $\tau_k = 0.25$	133.6	—	2.3	—
$\partial g^*\|_{T^*\text{constant}} < 0$, $g^* = 0.20$	−132.7	—	55.1	—
$\partial\sigma\|_{T^*\text{constant}} < 0$, $\sigma = 0.04$	1.1	—	0.2	—
Balanced budget policy				
Benchmark set[b]	154.5	0[c]	—	—
$\partial\tau_{k\|BB,do=0} < 0$	150.1	−8.0[c]	2.3	2.1[d]
$\partial g^*\|_{BB,do=0} > 0$	148.7	−10.6[c]	−68.3	−45.1[d]
$\partial\sigma\|_{BB,do=0} < 0$	627.9	869.4[c]	39.9	13.0[d]

[a] First column is percent change from benchmark of constant T^* policy; welfare is percent change from benchmark of constant T^* policy.

[b] Benchmark set of parameter values: $h = 10.0$; $\delta = 0.04$; $\alpha = 0.1$; $\tau_k = 0.30$; $g^* = 0.25$; $\gamma = 0.025$; $\sigma = 0.04125$; $k_o = 10.0$; $M_o = 0.16$; $B_o = 0.5$; which imply $\alpha k_o = 1$.

[c] Percent change from benchmark of balanced budget policy.

[d] Endogenous choices of $\{\partial\tau_k < 0, \partial g^* > 0, \partial\sigma < 0\}$ to satisfy Eqs. (18b–d) for the constant T^* policy; indicate percent change from benchmark of constant T^* policy.

the effects of arbitrary marginal cuts in each of the policy instruments, $\{\tau_k, g^*, \sigma\}$. The columns in Table 1 are organized with the first column for $V(T)$ denoting comparisons to the benchmark of constant T^* policy and the second column denoting comparisons to the benchmark of balanced budget (BB) policy. The third and fourth columns are welfare, Z, and welfare in the case where the long-term constraint shown in Eq. (18a) binds, $Z|_{LC}$, for the constant T^* policy. These welfare changes denote changes in the stocks evaluated by Eq. (12). Thus the first part of the first column and the first part of the second column represent the comparative statics expressed in Eq. (16 and 17), respectively, whereas the other parts represent the comparisons across regimes.

The first general comment regards dynamic scoring. With this set of plausible parameter values, dynamic scoring does occur with the implemented balanced budget policy, but not with the constant T^* policy. In the constant T^* policy, one would need necessarily a much larger rate of time preference relative to the rate of growth of money for dynamic scoring induced by the inflation tax and price level effects to occur in this setting.

The effects of government spending cuts give the highest welfare gain, 55.1% in the constant T^* case, and the highest welfare loss, (−)68.3%, when the spending is endogenously increased to balance the budget. The tax liability of the private sector decreases by 132.7% in the constant T^* policy and increases by 148.7% in the balanced budget policy relative to the benchmark of constant T^* policy (when the spending is endogenously increased to balance the budget).

A cut in the capital income tax yields a modest welfare gain, approximately 2.3%, with the largest increase in the lump sum tax liability of 133.6%. The balanced budget

policy is also achieved with an endogenous decrease in the capital income tax. The present value of taxes of the private sector increases up to 150.1% when compared with the constant T^* policy, but welfare increases by 2.3%.

The deflationary policy involves very small to significant welfare gains when compared with the other two policy instruments, however, the effect on the tax liability is more interesting. The tax liability of the private sector increases by 1.1% in the constant T^* policy relative to benchmark, but when the inflation tax is chosen endogenously to balance the budget, it involves turning it into a subsidy that gives the largest increase in the tax liability.

This table sheds light on the trade-off between the level of long-term tax liability versus the welfare of the private agent. The best policy to pursue may depend on the relative weights the policy maker attaches to these different objectives. If all the weight is attached to decreasing the tax liability of the private sector with no weight attached to the welfare of the private agent, there are some choices: (1) increasing the capital income tax or the inflation tax with a constant T^* policy or a balanced budget policy with any of the instruments achieves the objective or (2) decreasing government spending. Alternatively, if no weight is attached to the long-term tax liability and all the weight is attached to welfare, the cut in government spending gives the largest welfare gains in both policy regimes seconded by the deflationary policy with the balanced budget and followed by the cut in the capital income tax in the constant T^* policy. However, the instrument that achieves both objectives simultaneously is government spending, therefore superior to the other two in this dimension.

The welfare cost or benefit of satisfying the long-term constraint of Eq. (18a), or $V(t) = 0$ in the constant T^* policy, is given by the quantity $Z|_{LC}$ in the last column of Table 1. These values are percent changes from the benchmark of constant T^* policy and indicate the specific instrument used to satisfy the constraint of Eq. (18a), as in Eqs. (18b–d). Hence if the capital income tax is lowered endogenously to satisfy the long-term constraint, the gain in welfare is almost the same relative to the endogenous capital income tax necessary to balance the budget: 2.1% in the former and 2.3% in the latter. When the inflation tax is used, the welfare effect is smaller, and when government spending is increased, the loss in welfare is also smaller. Thus both the balanced budget policy along the balanced growth path and the constant T^* policy that satisfies the long-term constraint of Eq. (18a) give a welfare loss only if they are achieved by an endogenous government spending increase.

The price level effect is gauged in Table 2. These are percentage-point deviations from the values in Table 1 when $\gamma \to 0$. In general, the absence of nominal price flexibility increases the tax liability of the private sector and welfare in the constant T^* and balanced budget policies. In the balanced budget policy without nominal price flexibility, welfare changes less with the endogenous decrease in the capital income tax rate and with the endogenous higher government spending. This is because in the balanced budget policy, the requirement that the primary deficit be zero involves smaller variations in the capital income tax or government spending in the absence of the inflation tax and price level effects. As a result, welfare varies less in both cases.

Table 2
The nominal price effect, $\gamma \to 0$[a]

	$V(T)$	$V(T)$	Z	$X\|_{LC}$
Constant T^* policy				
Benchmark set	—	—	—	—
$\partial\tau_{k\|T^*\text{constant}} < 0, \tau_k = 0.25$	33.1	—	0.5	—
$\partial g^*\|_{T^*\text{constant}} < 0, g^* = 0.20$	34.0	—	11.7	—
Balanced budget policy				
Benchmark set	—	—	—	—
$\partial\tau_{k\|BB,do\equiv0} < 0$	16.6	0	0.5	0.2
$\partial g^*\|_{BB,do\equiv0} > 0$	18.0	0	5.2	−1.9

  [a] Percentage points from Table 1.

The role of nominal assets is seen to be important for both the tax liability and welfare. The range of variability is between an increment of 33.1% in the value of the tax liability for the cut in the capital income tax in the constant T^* policy to an additional 34.0% in the value of the tax liability for the cut in government spending. For welfare with the additional long-term constraint, it ranges from a gain of 0.2% to a loss of (−)1.9% in the balanced budget policy with capital income tax and government spending, respectively. In the case of the balanced budget policy, the present value becomes the initial real outstanding debt such that the absence of nominal price flexibility makes no difference (the second column) and the possibility of dynamic scoring disappears.

In general from Tables 1 and 2, the balanced budget policy is the one that consistently deteriorates the lump sum tax liabilities because the initial situation is one of primary surplus and lump sum tax credit. There would be a modest welfare loss if the capital income tax had to be endogenously raised to balance the primary deficit or to satisfy the long-term constraint, and the alternative welfare gain would be larger with an endogenous cut in government spending.

To summarize, changes in government spending provide variations in welfare and in the tax liability of the private sector that make it the most attractive policy instrument, more so in the constant T^* policy relative to the balanced budget policy. The endogenous cut in the capital income tax necessary to implement the balanced budget policy or to satisfy the long-term constraint gives modest welfare gains. The more dramatic increase in the long-term tax liability of the private sector is given by the endogenous balanced budget deflationary policy, but the welfare gain is mild.

6. Conclusions

The role of intertemporal budget policies in guaranteeing intertemporal government budget solvency has been assessed in an endogenous growth model with nominal assets where growth is driven by a Tobin (1969) q-type effect. The main results of the paper regard: (1) the relative rankings of the alternative policies and policy

instruments in terms of the tax liabilities of the private sector necessary to guarantee intertemporal solvency and in terms of the welfare of the representative agent, and (2) the role of nominal assets.

In particular, both the balanced budget policy along the balanced growth path or the policy that satisfies a long-term balance budget constraint are welfare enhancing if they are achieved by an endogenous capital income tax cut or by a deflationary policy; in the case of a raise in government spending, they have a large welfare cost. However, the balanced budget policy is the one that consistently deteriorates the lump sum tax liabilities when compared with the constant T^* policy, because the initial position is of tax credit. Increases in non–utility-enhancing government spending provide welfare losses and increases in the tax liability of the private sector that make it the less attractive policy instrument. Although dynamic scoring occurs in the balanced budget policy, in the constant T^* policy, one would need a significantly larger rate of time preference relative to the rate of growth of money for dynamic scoring induced by the inflation tax and price level effects to occur in this setting. The role of nominal assets is seen to be of critical importance for dynamic scoring, and nominal price flexibility accounts for a range of variation in the tax liability and welfare of up to 34 and 11.7 percentage points, respectively.

An extension of this framework for an open economy may prove fruitful because growth rates within the nation may diverge and government policy may have current account effects.

Notes

1. The related papers in this strand of the literature that introduce money are, among others, De Gregorio (1993), Jones and Manuelli (1995), Palivos and Yip (1995), and Pecorino (1995).
2. Dynamic scoring is meant as interactions of growth rates and tax bases such that lower tax rates or higher government spending may lead ultimately to lower long-term tax liabilities of the private sector. The paper by Bruce and Turnovsky (1999) shows this dynamic scoring possibility to depend critically on the elasticity of intertemporal substitution (EIS) in the one-sector endogenous growth model, whereas Bianconi (1996) extends their framework to a multisector endogenous growth model and shows that this effect is independent of the EIS in the multisector model. For analytical tractability, the results here are obtained for the case of unitary EIS [see, e.g., Beaudry & Van Wincoop (1996) for evidence in favor of the unitary EIS]. Hence, in this paper, dynamic scoring

possibilities are shown to depend more critically on the presence of nominal assets, given the unitary EIS.

3. Because the government debt valuation is separated from the solution of the real and nominal allocations in Eq. (1), a change in a tax rate on the return of government bonds does not change its after-tax rate of return, thus we do not consider a tax rate on the return on government bonds here.

4. Formally, this would imply $m \rightarrow 0$ or that the price level approaches infinity. The marginal implications for the equilibrium are in fact equivalent to a real model without nominal assets, when variables are properly transformed.

5. Whether there are nominal assets in the model is irrelevant for the existence of a q-growth path, say a model without money has the exact same real side determined by Eqs. (7a–e), see, for example, Turnovsky (1996a,b).

6. In the simulations below, the growth rate effect always dominates and $\partial Z/\partial \tau_k < 0$.

7. The qualitative marginal effects of the policy instruments $\{\tau_k, g^*, \sigma\}$ on $b(0) = B_o/P(0)$ are identical to the effects on $m(0) = M_o/P(0)$ given in section 2, above.

8. Authors such as Auerbach (1994) and Bruce and Turnovsky (1995) use some variants of this measure to evaluate the intertemporal solvency of the government; see also Bianconi (1996).

9. The condition $V(T) = 0$ has been used by Bruce and Turnovsky (1995) as a measure of long-term intertemporal balance for the government; see also Bianconi (1996).

10. See, for example, Beaudry and Guay (1996) for an estimation of the parameter h in this model. Their estimates range from 8 to 17. I have calibrated the value of $h = 10$ in my model to obtain a steady growth rate of 2%.

References

Auerbach, A. J. (1994). The U.S. fiscal problem: where we are, how we got here, and where we're going. In S. Fischer and J. J. Rotemberg (Eds.), *NBER Macroeconomics Annual 1994* (pp. 141–175). Cambridge, MA: MIT Press.

Barro, R. J., & Sala-i-Martin, X. (1992). Public finance in models of economic growth. *Rev Econ Stud 59*, 645–661.

Beaudry, P., & van Wincoop, E. (1996). The intertemporal elasticity of substitution: an exploration using U.S. Panel of State data. *Economica 63*, 495–512.

Beaudry, P., & Guay, A. (1996). What do interest rates reveal about the functioning of real business cycle models? *J Econ Dynam Contr 20*, 1661–1682.

Bianconi, M. (1996). The effects of alternative fiscal policies on the intertemporal government budget constraint. Manuscript, Department of Economics, Tufts University, September.

Bruce, N., & Turnovsky, S. J. (1999). Budget policies and the growth rate: 'dynamic scoring' of long-run government balance. *J Money, Credit, Banking*, forthcoming.

De Gregorio, J. (1993). Inflation, taxation, and long run growth. *J Monetary Econ 31*, 271–298.

Hayashi, F. (1982). Tobin's marginal q and average q: a neoclassical interpretation. *Econometrica 50*, 213–224.

Jones, L., & Manuelli, R. (1995). Growth and the effects of inflation. *J Econ Dynam Contr 19*, 1405–1428.

Palivos, T., & Yip, C. K. (1995). Government expenditure financing in an endogenous growth model: a comparison. *J Money, Credit, Banking 27*, Part I, 1159–1178.

Pecorino, P. (1995). The optimal rate of inflation when capital is taxed. *J Macroecon*, forthcoming.

Rebelo, S. (1991). Long run policy analysis and long run growth. *J Polit Econ 99*, 500–521.

Sidrausky, M. (1967). Rational choice and patterns of growth in a monetary economy. *Am Econ Rev Papers Proc 57*, 534–544.

Tobin, J. (1969). A general equilibrium approach to monetary theory. *J Money, Credit, Banking 1*, 15–29.

Turnovsky, S. J. (1996a). Fiscal policy, adjustment costs, and endogenous growth. *Oxford Econ Papers 48*, 361–381.

Turnovsky, S. J. (1996b). Fiscal policy, growth and macroeconomic performance in a small open economy. *J Int Econ 40*, 41–66.