



Private information, growth, and asset prices with stochastic disturbances

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Abstract

We introduce both idiosyncratic and aggregate shocks in an endogenous growth model with endogenous partial insurance to the idiosyncratic shock. Aggregate uncertainty introduces an additional channel that can play an important role in determining the effects of private information on expected growth and asset prices. We show the impact of aggregate and idiosyncratic shocks on expected growth and on the variability of individual quantities and asset prices.

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1. Introduction

The last 10 to 15 years witnessed an increasing interest in the area of economic growth from the point of view of the neoclassical framework. The new interest emerged first from contributions in the 1980s that essentially directed this area to a second generation of growth models where either production externalities or human capital accumulation delivered growth endogenously. Parallel to this development, there has been an increased interest in the study of general equilibrium with informational asymmetries.¹

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¹ See Barro and Sala-i-Martin (1995), for a recent survey on economic growth, and the seminal contributions of Atkeson and Lucas (1992) and Prescott and Townsend (1984) on asymmetric information.

This article presents analytical and quantitative results in a simple general equilibrium framework with endogenous growth and potential private information about the productivity of individuals. In particular, growth is driven by a stochastic *Ak*-type technology as discussed in [Rebelo \(1991\)](#). The potential private information is about the individual productivity in operating a given stock of capital as in the recent contribution of [Khan and Ravikumar \(1997\)](#). The main contribution here is to introduce idiosyncratic and aggregate uncertainty, but the individual has no access to perfect capital markets. The extent of insurance against the idiosyncratic shock is endogenously determined by an efficient long-term contract with intermediaries. Risk-averse individuals would like to fully insure against idiosyncratic shocks but private information prevents them to do so. Thus, we analyze the effects on growth and asset prices of private versus full information of the idiosyncratic shock, in an economy with capital accumulation and aggregate uncertainty.²

A main contribution here is to explore the fact that the correlation between individual and aggregate productivity can be plausibly either positive or negative. We show that depending on the sign of this correlation, the effect of idiosyncratic risk on growth and asset prices can vary substantially. In addition, we show that the effect on individual variability, relative to aggregate variability, is a function of how aggregate uncertainty impacts on the probability distribution of the idiosyncratic shock. If the probability of high individual productivity is larger when the good aggregate state occurs, the variability of individual consumption is larger. However, if the probability of high individual productivity is smaller when the good aggregate state occurs, the variability of individual consumption is smaller.

In another dimension, [Khan and Ravikumar \(1997\)](#), in a similar model without aggregate uncertainty, have shown that private information reduces expected growth, and the effects are quantitatively small. Our result is that adding aggregate uncertainty introduces an additional channel that can mitigate, but never reverse, the negative distortionary effect of private information on expected growth. In addition, we show that the growth effects of individual private information with aggregate uncertainty may be larger than without aggregate uncertainty.

One of the motivations of this study relates to the well-known failure of the representative agent paradigm to cope with the fact that the variability of individual consumption and income is much larger than the variability of aggregate per individual consumption and income (e.g., [Deaton, 1991](#); [Deaton, 1992](#); [Pischke, 1995](#)). In a model without private information and consequent full insurance to the idiosyncratic shock, individual allocations are identical to aggregate per individual allocations, i.e., the variability of individual quantities is identical to the variability of aggregate per individual quantities as in the representative agent framework. However, [Pischke \(1995\)](#) shows empirically that the variability of idiosyncratic income and consumption is about 40 times larger than the variability of aggregate per individual income and consumption. Thus, a serious model of individual heterogeneity must include discrep-

² See, for example, [Phelan \(1994\)](#) for a discussion of aggregate shocks and incentives in an overlapping generations framework. The paper by [Den Haan \(1997\)](#) discusses related computational and calibration issues in general equilibrium models with heterogeneous agents and aggregate shocks.

ancies between individual and aggregate allocations.³ Although a model with idiosyncratic uncertainty and private information can generate differences in individual variability versus aggregate per individual variability, an important and interesting question is whether private information with idiosyncratic and aggregate shocks can generate significant differences in individual variability versus aggregate per individual variability.

In our model, there is no private or public insurance mechanism available for aggregate shocks. However, an important question is whether these insurance arrangements would be desirable. [Attanasio and Rios-Rull \(1999\)](#) examine the role of public insurance to aggregate shocks in an endowment economy with partial insurance to idiosyncratic shocks. They find that the provision of public insurance can have distortionary effects on private mechanisms. In the context of our growth model, we find that aggregate shocks can mitigate the negative growth effects of private information. Hence, under private information, the provision of public insurance can have detrimental effects on expected growth.

Finally, we discuss the effects on asset prices and excess returns exploring the property that, in the growth framework, the marginal rate of substitution is a function of the growth factor only, not levels.⁴ As expected, in the case of logarithmic utility, adding private information in the general equilibrium asset pricing framework may increase the variability of the marginal rate of substitution thus increasing the excess return (e.g., [Heaton & Lucas, 1992](#)).

The article is organized as follows. Section 2 presents the basic structure while Section 3 presents the solution for the optimal contract. Section 4 is the core of the article where the alternative shocks and information arrangements are analyzed and comparisons and numerical examples are presented. Section 5 concludes.

2. Basic structure

This is a one-good model with a large number of individual households. All variables are for each individual unless otherwise noted. Time is discrete and a prime next to a variable denotes its next period value. [Fig. 1](#), which will be recalled throughout, presents a sketch of the timings and activities in the model. It is important to emphasize that the long-term contract characterized in this article is only contingent on the initial state, but because we

³ See, for example, the survey of [Rios-Rull \(1995\)](#), the discussion of [Carroll \(2000\)](#), and the applications by [Atkeson and Lucas \(1992\)](#), [Den Haan \(1996, 1997\)](#), [Heaton and Lucas \(1992\)](#), [Kahn \(1990\)](#), [Kocherlakota \(1998\)](#), and [Krusell and Smith \(1998\)](#) among others. [Pischke \(1995\)](#) associates the discrepancy between individual and aggregate variability with the possibility that individuals do not consider aggregate uncertainty in their decision-making process. This claim is in part inspired by the analytical results of [Goodfriend \(1992\)](#) who argue that individuals are not able to observe the current aggregate state, but only with a time lag. Here, I assume that the current aggregate uncertainty is known in the beginning of the current period.

⁴ See, e.g., [Brock \(1982\)](#) and [Cochrane \(1991\)](#) for discussions of asset prices in a production framework. The point that the marginal rate of substitution depends on growth is emphasized in [Mehra and Prescott \(1985\)](#) for an endowment economy. [Hansen and Jagannathan \(1991\)](#) examine the role of the marginal rate of substitution in asset pricing models.

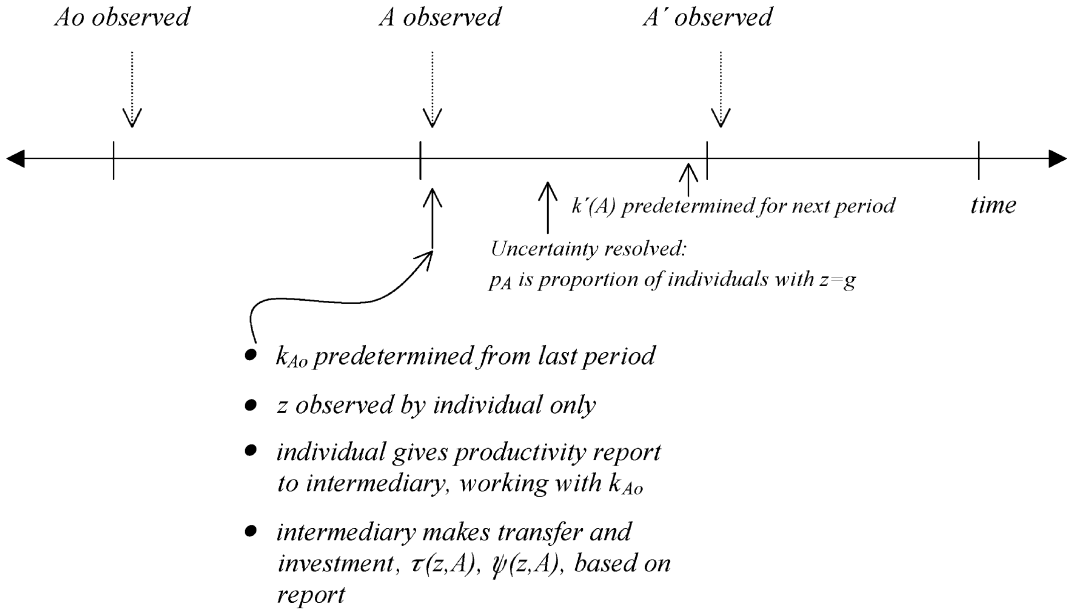


Fig. 1. Timing and activities.

derive a separating equilibrium along a stochastic balanced growth path, in this equilibrium, variables evolve sequentially as in Fig. 1.

We start with the production side. Current output, per quantity of capital, for an individual is denoted by the linear function

$$y(z,A) = z + A \quad (1)$$

where $y(z,A)$ is output deflated by the initial level of capital k_{A_0} that is predetermined from the last period as a function of the aggregate state of technology last period A_0 , used for production in the current period (e.g., Fig. 1). Capital is assumed to fully depreciate every period. A is the current period aggregate state of technology, assumed to be *i.i.d.*, with probability function

$$A = G \text{ with probability } \pi,$$

$$A = B < G \text{ with probability } 1 - \pi$$

where $\pi \in [0,1]$. The unconditional mean of A is assumed to satisfy, $E_A[A] \geq 1$, where E_x is the expectation operator over X . The idiosyncratic component of the technology, z , is assumed to be independently distributed, and may be individual's private information. The probability function of z is

$$z = g \text{ with probability } p_A,$$

$$z = b < g \text{ with probability } 1 - p_A$$

with $p_A \in [0,1]$. Hence, the probability function of z depends on the aggregate state A , and for each A , the probability function may shift. The conditional mean, denoted $\mu(A)$, is

$$E_z[z] = p_A g + (1 - p_A) b \equiv \mu(A) \geq 0.$$

The key issue here is that the effect of aggregate risk on the probability distribution of individual productivity can plausibly make the slope of the function $\mu(A)$ be positive or negative (or zero). For example, in the good aggregate state, G , the probability of high individual productivity, p_G , may increase when individuals are willing to be more efficient, implying that μ is increasing in A . However, the probability of high individual productivity, p_G , may decrease when individuals are willing to be less efficient, given G , implying alternatively that μ is decreasing in A .⁵

Given Eq. (1), average aggregate output per individual, deflated by initial capital, is

$$E_{z,A}[z + A] = \pi[\mu(G) + G] + (1 - \pi)[\mu(B) + B].$$

Individuals are assumed to be risk-averse, with average logarithmic utility

$$v(C) = (1 - \beta) \log C \tag{2}$$

where $\beta \in (0,1)$ denotes the subjective discount factor assumed to be identical across individuals and C is consumption.⁶

3. Private information, incentives, and general equilibrium

The idiosyncratic component z is the individual's private information so that a revelation mechanism has to be designed. We proceed by designing a mechanism based on a simple long-term principal–agent relationship as in [Khan and Ravikumar \(1997\)](#) (e.g., [Townsend, 1982](#)). There are several risk-neutral intermediaries operating competitively and each individual enters a long-term relationship with one of the intermediaries. A typical contract between an individual and an intermediary specifies: (i) a contingent current transfer, per quantity of capital, from the intermediary to the individual denoted by the function $\tau(z,A)$;

⁵ On another dimension, the risk of unemployment may be higher in a recession than in a boom. So, it would be plausible to assume that the variability of individual quantities is higher in the bad aggregate state than in the good as if downside uncertainty matters more (see, e.g., [Heaton & Lucas, 1992](#), on this point).

⁶ Following [Khan and Ravikumar \(1997\)](#), we explore the homogeneity properties of the functions above (see e.g., [Alvarez & Stokey, 1998](#), for a discussion), to express the model per quantity of initial capital, thus deflating the relevant variables by k_{A_0} . This will ultimately impose stationarity in the relevant variables as is usual in balanced growth models.

(ii) contingent current investment, per quantity of capital, from the intermediary to the individual denoted by the function $\psi(z,A)$ that is identical to the growth factor of the capital stock. The timing presented in Fig. 1 shows that all observe the current aggregate state in the beginning of the period. Next, the individual provides a report to the intermediary and the intermediary decides on the appropriate transfer and investment to the individual.⁷ Afterwards, uncertainty is resolved and the probability function of the idiosyncratic shock yields the proportion of individuals with respective idiosyncratic components so that the average capital stock is predetermined for next period.

Hence, the current contingent expenditure, deflated by the capital stock, for the risk-neutral intermediary amounts to

$$\tau(z,A) + \psi(z,A), \text{ each } A, z. \quad (3)$$

Individual current contingent consumption, $c(z,A) \equiv C(z,A,A_0)/k_{A_0}$, then consists of production plus transfers, or

$$c(z,A) = z + A + \tau(z,A), \text{ each } A, z. \quad (4)$$

We let U' be the current expected discounted lifetime utility entitlement starting from next period onwards, with current full commitment to z , and define a state variable,⁸

$$s' \equiv U' - \log k'.$$

Considering the lifetime utility entitlement deflated by the capital stock imposes stationarity in the state variable, as it does in all other variables of the balanced growth model, so that $s' = s$. Thus, using the definition of current investment, we have

$$\begin{aligned} s'(z',A') &= U'(z,A',A,A_0) - \log k'(z,A,A_0) \\ &= U'(z,A',A,A_0) - \log \psi(z,A) - \log k_{A_0} = s(z',A') \end{aligned} \quad (5)$$

and the linear combination of the lifetime utility entitlement U and $\log k$ is stationary, i.e., s is stationary.

⁷ The current transfer is $\tau(z,A) \equiv T(z,A,A_0)/k_{A_0}$ where T is the transfer and current investment is $\psi(z,A) \equiv k'(z,A,A_0)/k_{A_0}$ with the usual assumption that it takes one period for capital to become available for use in production. All ratios to the initial capital stock are stationary and independent of k_{A_0} because of the linear technology. For a recent discussion on the issue of the correlation between investment and growth in the Ak type of model, see McGrattan (1998). The report is assumed to be free of any cost (see, e.g., Prescott, 2001, for models of costly reporting as an action).

⁸ For example, in the simple two-period case, $U' \equiv (1-\beta)\beta \log C'$. Khan and Ravikumar (1997) also consider the more general case of isoelastic preferences, where the state variable becomes $U' - (1-\sigma)\log k'$, where σ is the coefficient of relative risk aversion.

Following Green (1987) and Khan and Ravikumar (1997),⁹ the revelation mechanism requires temporary incentive compatibility constraints of the form:

(i) For an individual, currently with $z=b$:

$$\begin{aligned} &\text{if } b + A + \tau(g, B) > 0, \text{ for all } A, \text{ then } (1 - \beta)\log(b + A + \tau(b, A)) \\ &+ \beta\log\psi(b, A) + \beta E_{A'}[s'(b, A')] \geq (1 - \beta)\log(b + A + \tau(g, A)) \\ &+ \beta\log\psi(g, A) + \beta E_{A'}[s'(g, A')], \text{ each } A; \end{aligned} \quad (6a)$$

(ii) For an individual with $z=g$:

$$\begin{aligned} &(1 - \beta)\log(g + A + \tau(g, A)) + \beta\log\psi(g, A) + \beta E_{A'}[s'(g, A')] \\ &\geq (1 - \beta)\log(g + A + \tau(b, A)) + \beta\log\psi(b, A) + \beta E_{A'}[s'(b, A')], \text{ each } A. \end{aligned} \quad (6b)$$

The constraint in Eq. (6a) implies that for all A , conditional on the current consumption of individual $z=b$ being strictly positive, when this individual misrepresents, i.e., $b+A+\tau(g,A)>0$, the lifetime utility obtained with truth telling, that is the LHS is

$$\begin{aligned} &(1 - \beta)\log(b + A + \tau(b, A)) + \beta E_{A'}[U'(b, A', A, A_0)] \\ &= (1 - \beta)\log(b + A + \tau(b, A)) + \beta E_{A'}[s'(b, A')] + \beta\log\psi(b, A) \end{aligned}$$

must be no less than the lifetime utility obtained with current misrepresentation and onwards, the RHS

$$(1 - \beta)\log(b + A + \tau(g, A)) + \beta E_{A'}[s'(g, A')] + \beta\log\psi(g, A).$$

Then, for individual $z=g$, for all A , $g+A+\tau(b,A)>0$ holds by (i), and Eq. (6b) requires that the lifetime utility obtained with truth telling, i.e., the LHS,

$$(1 - \beta)\log(g + A + \tau(g, A)) + \beta E_{A'}[s'(g, A')] + \beta\log\psi(g, A),$$

must be no less than the lifetime utility obtained with current misrepresentation and onwards, i.e., the RHS

$$(1 - \beta)\log(g + A + \tau(b, A)) + \beta E_{A'}[s'(b, A')] + \beta\log\psi(b, A).$$

⁹ Since Green's (1987) contribution, other papers followed in this tradition including Green and Oh (1991), Marcet and Marimon (1992), and Thomas and Worrall (1990).

The participation constraint for all individuals is given by

$$s(z, A) \leq E_z[(1 - \beta)\log(z + A + \tau(z, A)) + \beta E_{A'}[s'(z', A') + \beta \log \psi(z, A)], \text{ each } A, \quad (7)$$

which states that for each A , the expected (over z) lifetime utility entering the contract can be no less than the initial expected lifetime utility.¹⁰

In addition, using the law of large numbers, we can average s' across all individuals (z) so that letting the population become large allows the idiosyncratic component of the state variable to vanish in equilibrium, i.e., $\lim_{i \rightarrow \infty} \int_{z'_i} s'(z'_i, A') f_{z'_i}(z'_i | A') dz'_i = s'(A') = s(A')$; where $f_{z'_i}(z'_i | A')$ is the probability distribution of the idiosyncratic shocks, conditional on A' , across all individuals.

For a characterization of the optimal contract, we use the dual approach: the principal solves an expenditure minimization problem whose solution yields the optimal contract (e.g., Green, 1987; Khan & Ravikumar, 1997). Hence, using Eq. (3), the optimal contract is the solution to Bellman's functional equation

$$W(s(A)) = \min E_z[\tau(z, A) + \psi(z, A)(1 + \rho E_{A'}[W(s'(A'))])], \text{ each } A \quad (8)$$

where the inner minimum is by choice of $\{\tau(z, A), \psi(z, A), s'(A')\}$ subject to Eqs. (6a), (6b), and (7), taking ρ , the discount factor among intermediaries as given, as well as the probability distributions of the shocks.¹¹

The solution for the optimal contract must provide the right incentive for each individual to reveal its type truthfully. In principle, deviations from truth telling allow individuals to consume extra hidden output. To avoid this, the optimal contract has the following characteristics: (i) incentive constraint (6a) never binds; (ii) incentive constraint (6b) binds with associated contingent Lagrange multipliers denoted $\lambda(A) \geq 0$; (iii) the participation constraint binds with associated contingent Lagrange multipliers denoted $\eta(A) > 0$. The saddle point for the appropriate Lagrangian function implies first-order necessary conditions for $\{\tau(z, A), \psi(z, A), s'(A'), \lambda(A), \eta(A)\}$, respectively, given by

$$p_A(g + A + \tau(g, A)) - (1 - \beta)(p_A \eta(A) + \lambda(A)) = 0, \text{ each } A \quad (9a)$$

$$(1 - p_A) - (1 - \beta)[((1 - p_A)\eta(A)/(b + A + \tau(b, A))) - [\lambda(A)/(g + A + \tau(b, A))]] = 0, \text{ each } A \quad (9b)$$

¹⁰ In the incentive compatibility constraints $\log k_{A_t}$ cancels out in both sides of Eqs. (6a) and (6b) and the participation constraint is obtained using the definition of the state variable.

¹¹ The Bellman equation in Eq. (8) is obtained assuming that the principal is risk-neutral, with net expenditure per quantity of capital $\tau(z, A) + \psi(z, A)$. Thus, the function W is the net expenditure per quantity of capital and it is strictly increasing, strictly monotonic, and convex (see, e.g., Khan & Ravikumar, 1997; Stokey, Lucas, & Prescott, 1989, for the standard results).

$$p_A \psi(g, A)(1 + \rho E_{A'}[W(s'(A'))]) - \beta(p_A \eta(A) + \lambda(A)) = 0, \text{ each } A \quad (9c)$$

$$(1 - p_A) \psi(b, A)(1 + \rho E_{A'}[W(s'(A'))]) - \beta((1 - p_A) \eta(A) - \lambda(A)) = 0, \text{ each } A \quad (9d)$$

$$p_A \psi(g, A) \rho E_{A'}[\partial W(s'(A')) / \partial s'] - \beta(p_A \eta(A) + \lambda(A)) = 0, \text{ each } A \quad (9e)$$

$$(1 - p_A) \psi(b, A) \rho E_{A'}[\partial W(s'(A')) / \partial s'] - \beta((1 - p_A) \eta(A) - \lambda(A)) = 0, \text{ each } A \quad (9f)$$

together with the constraints (6b) and (7) holding with equality for each A . The set of first-order conditions yield a total of 16 equations in the 16 unknowns $\{\tau(z, A), \psi(z, A), s'(A'), \lambda(A), \eta(A)\}$. The envelope condition (Benveniste–Scheinkman formula) yields:

$$\partial W(s(A)) / \partial s = \eta(A), \text{ each } A. \quad (10)$$

The Lagrange multiplier $\eta(A) > 0$ represents the marginal cost of the initial lifetime utility per unit of capital. The other multiplier $\lambda(A) \geq 0$ plays an important role in the analysis. It represents the marginal cost, in terms of utils, for an individual with $z=g$ to falsely report $z=b$ and receive transfer $\tau(b, A)$. Hence, $\lambda(A)$ represents the marginal efficiency of the contract. If $\lambda(A)=0$, there is no binding commitment to truth telling and full insurance to the idiosyncratic shock (full risk sharing) is provided by the principal. However, this first best solution does not give any incentive for truth telling when there is private information, so that we observe the usual trade off between risk sharing and incentives. As $\lambda(A) > 0$ increases, it gives the value of the contract in terms of the cost of misrepresenting. In particular, the saddle point for the Lagrangian function of Eq. (8) yields the maximum $\lambda(A)$ that minimizes expenditures. Hence, the larger $\lambda(A)$, the more efficient the contract is in terms of exploring the trade off between (partial) risk sharing and incentives.

The optimal contract characterized in Eqs. (9a)–(9f) is a classic separating equilibrium contract. It gives the right incentive for the low-productivity individual to reveal truthfully, while making the high-productivity individual indifferent. The low-productivity individual obtains a small surplus, which induces truth telling, whereas the high-productivity individual has no incentive to deviate from truth telling.

3.1. General equilibrium and asset prices

Perfect competition among intermediaries implies that expenditure will be driven to a minimum or

$$W(s(A)) = \min E_z[\tau(z, A) + \psi(z, A)(1 + \rho E_{A'}[W(s'(A'))])] = 0, \text{ each } A. \quad (11)$$

Any individual with initial capital k , expected lifetime utility U , and marginal product $z+A$ has current consumption, per quantity of capital, given by Eq. (4), transfer, per quantity of capital, determined by Eqs. (9a)–(9f), investment, per quantity of capital, also determined by Eqs. (9a)–(9f), and output, per quantity of capital, determined by Eq. (1). Average per capita aggregate quantities can then be computed along the stochastic balanced growth path subject to the economy-wide resources constraint holding for each current aggregate state, that is

$$\int_k E_z[(z+A)k_{A_0} - (z+A+\tau(z,A))k_{A_0} - \psi(z,A)k_{A_0}]f_k(k|A_0)dk = 0, \text{ each } A \quad (12)$$

where $f_k(k|A_0)$ is the probability distribution of the current capital stock, conditional on A_0 , across individuals. From expression (12), in general equilibrium, ρ has to be such that

$$E_z[\tau(z,A) + \psi(z,A)] = 0, \text{ each } A \quad (13)$$

or average individual aggregate saving ($=E_z[-\tau(z,A)]$) equals average individual investment ($=E_z[\psi(z,A)]$).¹² Using Eqs. (11) and (13), note that

$$E_{A'}[W(s'(A'))] = 0 \quad (14)$$

so that it confirms the stationarity of s . The contract is symmetric across all individuals. Using the first-order conditions for $\psi(z,A)$ and $s'(A')$, i.e., Eqs. (9c)–(9f), with Eqs. (10), (13), and (14) yields

$$\rho = E_{A'}[1/\eta(A')], \quad (15)$$

the risk-free discount factor among intermediaries, which closes the solution for the model.

In fact, ρ is the price of one unit of consumption in every state next period. To see this, note that the marginal rate of substitution in the growth framework (here with logarithmic utility) is

$$\text{MRS}(z',A') = 1/\psi(z',A'), \text{ each } A', z', \quad (16)$$

i.e., it only depends on the growth factor not levels. Hence, we can explore this property in studying asset prices (e.g., Hansen & Jagannathan, 1991; Mehra & Prescott, 1985). From the first-order necessary conditions for $\psi(z,A)$, Eqs. (9c) and (9d) using Eq. (14) yields

$$E_z[\psi(z,A)] = \beta\eta(A), \text{ each } A \quad (17)$$

so that the asset pricing formula for the one period risk-free asset, $\beta E_{z',A'}[\text{MRS}(z',A')]$, can be applied to deliver Eq. (15).

¹² The procedure of imposing an aggregate resources constraint to obtain the market interest rate is due to Atkeson and Lucas (1992). Although the long-term contract is contingent on the initial state only, the general equilibrium guarantees that the contract will be fulfilled period by period as well as in Khan and Ravikumar (1997).

Let $q(z,A)$ denote the price of a claim, among intermediaries, to all future risky dividends from the technology, i.e., the price of equity in this economy. For the logarithmic utility case examined here, it must solve the recursive formula

$$q(z,A) = \beta E_{A',z}[\{1/\psi(z',A')\}(q(z',A') + y(z',A'))], \text{ each } A,z \quad (18)$$

which, as in Mehra and Prescott (1985), yields a system of $A \times z$ linear equations in the $q(z,A)$ unknowns. Since shocks are *i.i.d.* and utility logarithmic, the stationary solution is

$$q(z,A) = q = \beta/(1 - \beta), \text{ all } A,z \quad (19)$$

and the excess return is a function of the discount factor β .¹³ Therefore, the price of equity is going to be Eq. (19) in all economies discussed below, so that we focus on the risk-free asset that relates to the marginal rate of substitution, which in turn is only a function of the growth factor.

To sum, the solution is consistent with on going growth of levels, and allocations per quantity of capital and prices, i.e., $\{\tau(z,A), \psi(z,A), s'(A'), \lambda(A), \eta(A), \rho\}$, all stationary. The optimal contract is offered to all with the right incentives for each type to reveal truthfully, i.e., a separating equilibrium. In particular, the intermediary provides surplus to the low-productivity individual to avoid making a larger investment in that individual. On the other hand, the high productivity is indifferent but has no incentive to deviate from truth telling.

4. Growth and asset prices with alternative arrangements

We proceed by examining alternative stochastic and informational structures and their impact on asset prices, growth, and variability.

4.1. Aggregate uncertainty only

Consider first the simplest case of no private information in the returns to capital with $z=0$, all z . Then, this economy is the Brock and Mirman (1972) economy with the Ak technology of Rebelo (1991), or Brock–Mirman meets endogenous growth. In particular, there is no discrepancy between aggregate and individual quantities, the typical representative agent framework. As usual, the individual cannot insure against aggregate risk, so that allocations are contingent on the aggregate state. The closed-form solution for the model is

¹³ The stationary result for the price of equity is obtained by assuming that $q(z,A)/y(z,a)=q(z',A')/y(z',A')=q$, constant. Atkeson and Lucas (1992) discuss the possibility of decentralizing efficient allocations using securities trade in the case of private information.

simple and obtained from the solution of Eqs. (9a)–(9f), with $\lambda(A)=0$, all A , and $z=0$, all z , yielding

$$\psi(A) = \beta A, \text{ each } A \quad (20a)$$

$$c(A) = (1 - \beta)A, \text{ each } A \quad (20b)$$

$$\tau(A) = -\beta A, \text{ each } A \quad (20c)$$

$$\rho = E_{A'}[1/A'] \quad (20d)$$

$$\text{MRS}(A') = 1/\beta A', \text{ each } A'. \quad (20e)$$

Thus, we have that

$$\psi(G) > \psi(B), \quad c(G) > c(B), \quad \tau(G) < \tau(B),$$

and ρ (or the MRS) depends on the variance of the aggregate disturbance. In equilibrium, consumption, growth (investment), and saving (negative transfers) are larger in the good aggregate state. If the variance of the aggregate shock, A , increases, by Jensen's inequality, ρ increases and the risk-free rate decreases, hence increasing the excess return, i.e., it implies higher variability of the expected marginal rate of substitution (e.g., [Hansen & Jagannathan, 1991](#)). However, aggregate and individual quantities have the same variability as in the representative agent case.

4.2. *Idiosyncratic shocks only with full information (heterogeneity only)*

Consider the case of no aggregate uncertainty in the returns to capital with $E[A]=A=1$ constant, and let there be no private information of the idiosyncratic shock so that z is fully observed by the intermediary. This economy is discussed in [Khan and Ravikumar \(1997\)](#) (see also [Marcet & Marimon, 1992](#)). There is no discrepancy between aggregate and individual quantities as in the representative agent case because the principal, who is risk-neutral, bears all the idiosyncratic risk, thus providing full insurance to the risk-averse individual. The closed-form solution for this economy is obtained from Eq. (8), with $\lambda(A)=0$, all A , and $E[A]=A=1$, yielding

$$\psi = \beta(\mu + 1) \quad (21a)$$

$$c = (1 - \beta)(\mu + 1) \quad (21b)$$

$$\tau(z) = -\beta(z + 1), \text{ each } z \quad (21c)$$

$$\rho = 1/(\mu + 1) \quad (21d)$$

$$\text{MRS} = 1/\beta(\mu + 1). \quad (21e)$$

where $\mu = pg + (1-p)b$. Thus, we have that

$$\psi(g) = \psi(b), \quad c(g) = c(b), \quad \tau(g) < \tau(b),$$

the full insurance (full risk sharing) of idiosyncratic risk solution. In this case, there is a full transfer, $\tau(b) > \tau(g)$, to the low-productivity individual to allow equality of consumption and investment across individuals.

4.3. Aggregate and idiosyncratic shocks with full information

Consider the case of aggregate and idiosyncratic uncertainty, but no private information of the idiosyncratic shock so that z is fully observed by the intermediary. Again, there is no discrepancy between aggregate and individual quantities as in the representative agent case because the principal, who is risk-neutral, bears all the risk of the individual uncertainty, thus providing full insurance to the idiosyncratic component of the risk-averse individual. However, the individual is not insured against aggregate shocks. The closed-form solution for this economy is obtained from Eqs. (9a)–(9f), with $\lambda(A) = 0$, all A , yielding

$$\psi(A) = \beta(\mu(A) + A), \quad \text{each } A \quad (22a)$$

$$c(A) = (1 - \beta)(\mu(A) + A), \quad \text{each } A \quad (22b)$$

$$\tau(z, A) = -\beta(z + A), \quad \text{each } A, z \quad (22c)$$

$$\rho = E_{A'}[1/(\mu(A') + A')], \quad (22d)$$

$$\text{MRS}(A') = 1/\beta(\mu(A') + A'), \quad \text{each } A'. \quad (22e)$$

Thus, we have that

$$\psi(g, A) = \psi(b, A), \quad c(g, A) = c(b, A), \quad \tau(g, A) < \tau(b, A), \quad \text{each } A$$

all contingent on the aggregate shock A . Full insurance (full risk sharing) for the idiosyncratic risk is provided by the principal, with full transfer contingent on A .

4.4. Idiosyncratic shocks only with private information

The three arrangements discussed so far have yielded allocations where the individual quantities are equal to the aggregate per individual quantities due to the provision of full insurance for idiosyncratic shocks. Consider now a case of no aggregate uncertainty in the returns to capital, or $E[A] = A = 1$ constant, with private information of the idiosyncratic shock so that z is not observed by the intermediary as in [Khan and Ravikumar \(1997\)](#). There is discrepancy between aggregate and individual quantities because the principal, who is risk-neutral, is not going to bear all the risk of the individual's uncertainty, thus providing only

partial insurance to the risk-averse individual. The private information requires a revelation mechanism to induce truth telling among heterogeneous individuals. The partial insurance mechanism is endogenously determined by the optimal contract (Eqs. (9a)–(9f)).¹⁴ This economy has the appealing property that individual allocations are more variable than aggregate per individual allocations as documented by [Deaton \(1991, 1992\)](#) and [Pischke \(1995\)](#). A closed-form solution for this case does not exist. The functional solution obtained from Eqs. (9a)–(9f), with $\lambda > 0$, so that the temporary incentive compatibility constraint (6b) holds with equality, and $E[A]=A=1$ constant, yields

$$E_z[\psi(z)] = \beta\eta \quad (23a)$$

$$E_z[c(z)] = \mu(A) + 1 - \beta\eta \quad (23b)$$

$$E_z[\tau(z)] = -\beta\eta \quad (23c)$$

$$\rho = \beta E_z[1/\psi'(z')] = 1/\eta \quad (23d)$$

$$\text{MRS} = E_z[1/\psi'(z')] = 1/\beta\eta \quad (23e)$$

where $\eta > 0$ is the Lagrange multiplier on Eq. (7) satisfying Eq. (10). [Kahn and Ravikumar \(1997\)](#) characterize the optimal contract obtaining

$$\psi(g) > \psi(b), \quad c(g) > c(b), \quad \tau(g) < \tau(b). \quad (24)$$

First, the high-productivity individual receives a higher investment thus can enjoy higher consumption, and receives a smaller transfer. Hence, we see from Eqs. (23a)–(23c) and (24) that a mean preserving spread of the distribution of the individual shock makes individual quantities more variable than aggregate per individual quantities. Also note that the price of the risk-free asset ρ (and MRS) varies inversely with the marginal cost of lifetime utility, η , and directly with the variance of z , thus improving excess returns.

In this case, different individuals have different consumption and investment bundles and the transfer scheme is endogenously partial since the optimal contract provides the right incentive for individuals to reveal their idiosyncratic productivity truthfully. The optimal contract generates a current transfer, in terms of the excess of production over consumption plus investment, from the high productivity to the low productivity, so that

$$b + 1 < c(b) + \psi(b) < c(g) + \psi(g) < g + 1, \quad \text{given } A = 1. \quad (25)$$

Under autarky, each would consume and invest out of its own productivity without net trades and each side of Eq. (25) would hold with equality; and under full risk sharing the differences would be fully traded so that $c(b)+\psi(b)=c(g)+\psi(g)$.

¹⁴ The issue of partial versus full risk sharing is also popular in the international finance literature (see, e.g., [Van Wincoop, 1999](#), and the references therein for a recent analysis).

The mechanism provides the right incentive for the low productivity to reveal truthfully without giving incentive for the high productivity to deviate from truth telling. Hence, the high-productivity individual receives higher consumption and investment whereas the low productivity receives lower consumption and investment. In this case, we can show that the marginal efficiency of the contract can be expressed as

$$\lambda = p(1 - p)(\psi(g) - \psi(b))/\beta \tag{26}$$

where $p_A=p$, for $A=1$ constant. Thus, the efficiency of the contract increases with the spread of $\psi(z)$, or the variance of z through the term $p(1-p)$, i.e., the variance of the one trial binomial. In this case, as the variability of z increases, the marginal cost of deviating from truth telling increases and the contract becomes more efficient in partially insuring the increased idiosyncratic risk.

4.5. Aggregate and idiosyncratic shocks with private information

The most general case is the one with aggregate and idiosyncratic uncertainty, and private information of the idiosyncratic shock so that z is not observable by the intermediary. There is discrepancy between aggregate and individual quantities with endogenous partial insurance of the idiosyncratic shock, however, as before, aggregate risk is systematic at the individual level. The closed-form solution for this economy does not exist, and the solution from Eqs. (9a)–(9f), with $\lambda(A)>0$, i.e., the temporary incentive compatibility constraint (6b) holding with equality, yields

$$E_z[\psi(z, A)] = \beta\eta(A), \text{ each } A \tag{27a}$$

$$E_z[c(z, A)] = \mu(A) + A - \beta\eta(A), \text{ each } A \tag{27b}$$

$$E_z[\tau(z, A)] = -\beta\eta(A), \text{ each } A \tag{27c}$$

$$\rho = \beta E_{z', A'}[1/\psi'(z', A')] = E_{A'}[1/\eta(A')] \tag{27d}$$

$$\text{MRS}(A') = E_{z'}[1/\psi'(z', A')] = 1/\beta\eta(A'), \text{ each } A' \tag{27e}$$

where $\eta(A)>0$ is the contingent Lagrange multiplier on Eq. (7) satisfying Eq. (10).

First, consider the case where $p_A=p$ for all A . Then, given the probability functions for the aggregate and idiosyncratic shocks, we have that

$$\psi(g, G) > \psi(g, B) \geq \psi(b, G) > \psi(b, B) \tag{28a}$$

$$c(g, G) > c(g, B) \geq c(b, G) > c(b, B) \tag{28b}$$

$$\tau(g, G) < \tau(g, B) \geq \tau(b, G) < \tau(b, B). \quad (28c)$$

Thus, by Eqs. (27a)–(27e) and Eqs. (28a)–(28c), the individual variability is enhanced by the superimposition of the aggregate uncertainty on the idiosyncratic shock relative to the absence of aggregate uncertainty. Again, the optimal contract generates a transfer of current production over consumption plus investment, from the high to the low productivity type, contingent on the aggregate state A , or

$$b + A < c(b, A) + \psi(b, A) < c(g, A) + \psi(g, A) < g + A, \text{ each } A.$$

We can show, using Eq. (28a), that

$$E_z[\psi(z, G)] - E_z[\psi(z, B)] = \beta(\eta(G) - \eta(B)) > 0, \quad (29)$$

implying that

$$\eta(G) > \eta(B)$$

and the marginal cost of lifetime utility is larger in the good aggregate state relative to the bad aggregate state because there is overall higher consumption and growth in the good aggregate state for all types. However,

$$\lambda(G) - \lambda(B) = (p(1-p)/\beta)(\{\psi(g, G) - \psi(b, G)\} - \{\psi(g, B) - \psi(b, B)\}) \geq 0 \quad (30)$$

implies that

$$\lambda(G) \geq \lambda(B).$$

Thus, by Eq. (30), the marginal cost of deviating from truth telling or the marginal efficiency of the contract may be higher or lower across aggregate states depending on the variability of the idiosyncratic shock across aggregate states. If there is more idiosyncratic variability in the good aggregate state, then $\lambda(G) > \lambda(B)$ and the contract is more efficient in that state and vice versa.

The price of the risk-free asset ρ (and MRS) varies directly with the variance of z and the variance of A . Thus, from the perspective of the excess returns, there is more variability in the MRS and thus an improvement in the excess return, relative to the absence of either z or A .

Next, consider the additional effects where p is contingent on A . First, the orderings $\psi(A, g) > \psi(A, b)$, $c(A, g) > c(A, b)$, and $\tau(A, g) < \tau(A, b)$ are preserved for all A . However, depending on how the probability function shifts with changes in A , we can end up with alternative rankings in Eqs. (28a)–(28c). First, examine the case when $p_G > p_B$, or the probability of the high productivity type is larger in the good aggregate state. By expressions (28a)–(28c) and (30), the individual variability is enhanced by the superimposition of the aggregate uncertainty on the idiosyncratic shock relative to the absence of aggregate uncertainty, or a positive correlation between z and A does not allow for diversification of risk.

However, when $p_G < p_B$, the probability of the high productivity type is smaller in the good aggregate state. In this case, an increase in the variance of z , given A , has to take into account the additional effect of A on p_A , which goes in the opposite direction. A negative correlation between z and A allows for some diversification of risk. Therefore, with aggregate uncertainty, the additional effect dampens the variability of individual quantities enough to make it smaller relative to the case of no aggregate uncertainty. Hence, the main result here is that with aggregate uncertainty, the variability of individual quantities is mitigated. The same is possible for the price of assets in this case. An increase in the variance of z , given A , can decrease the variability in the MRS and thus lower the excess return relative to the case of no aggregate uncertainty. In effect, under imperfect risk sharing, a negative correlation between z and A , reduces the variability of individual quantities.

4.6. Comparisons and simulations

Table 1 presents a summary of the results in the alternative arrangements for the expected (over z) growth factor, the discount factor (price of risk-free asset for intermediaries) and the marginal cost of deviations from truth telling or the marginal efficiency of the contract. As seen above, in the case where $p_A = p$ for all A , the expected value of the growth factor with respect to the aggregate shock, $E_{z,A}[\psi(z,A)]$, depends on the probability distribution of z and A . But, if p changes with the aggregate shock, then there is the additional channel where the expected growth factor is sensitive to the variability of both z and A . Similarly, for the price of the risk-free asset, it depends on the probability distribution of z and A , and the additional channel if p changes with the aggregate shock. The marginal efficiency of the contract, $\lambda(A)$, depends on the variability of the growth factor and $p_A(1-p_A)$, which is the variability of the one trial binomial for the idiosyncratic shock.

We can show, using the Eqs. (9a)–(9f) and the equilibrium condition (13) that

$$-\lambda(A)(1 - \beta)(1 - \{(b + A + \tau(b,A))/(g + A + \tau(b,A))\}) = \eta(A) - \mu(A) + A < 0, \text{ each } A.$$

Table 1
Growth and prices with alternative arrangements contingent on aggregate state

	(A) Aggregate uncertainty only	(B) Idiosyncratic shocks only with full information	(C) Aggregate and idiosyncratic shocks with full information	(D) Idiosyncratic shocks only with private information	(E) Aggregate and idiosyncratic shocks with private information
$E_z[\psi(z,A)]$	βA	$\beta \times (E_z[z]+A)$	$\beta \times E_z[z+A]$	$\beta \eta$	$\beta \eta(A)$
ρ	$E_A[A'^{-1}]$	$(E_z[z'+A']^{-1})$	$E_A[E_z[z'+A']^{-1}]$	η^{-1}	$E_A[\eta(A')^{-1}]$
$\lambda(A)$	0	0	0	$p(1-p) \times (\psi(g) - \psi(b)) \times \beta^{-1}$	$p_A(1-p_A) \times (\psi(g,A) - \psi(b,A)) \times \beta^{-1}$

Hence, we can establish from Table 1, columns B and D and columns C and E, that

$$E_z[\psi(z, A)] \mid \text{full information} > E_z[\psi(z, A)] \mid \text{private information, each } A, \quad (31a)$$

$$\rho \mid \text{full information} < \rho \mid \text{private information} \quad (31b)$$

so that, private information reduces the contingent average (over z) growth factor for each aggregate state. This is one of the main results of Kahn and Ravikumar (1997). Taking into account the additional channel where p depends on A , then

$$E_{z,A}[\psi(z, A)] \mid \text{full information} > E_{z,A}[\psi(z, A)] \mid \text{private information} \quad (32a)$$

$$\rho \mid \text{full information} < \rho \mid \text{private information} \quad (32b)$$

so that private information reduces the average (over z and A) growth factor and increases the discount factor. However, the key result here is that aggregate uncertainty mitigates the effect of private information, so that

$$\begin{aligned} E_z[\psi(z, A)] \mid \text{full information} - E_z[\psi(z, A)] \mid \text{private information} &\geq \\ E_{z,A}[\psi(z, A)] \mid \text{full information} - E_{z,A}[\psi(z, A)] \mid \text{private information}. &\quad (33) \end{aligned}$$

The gap between expected growth is smaller when there is aggregate uncertainty. Fig. 2 shows the RHS of Eq. (33) as a function of p_A . Expected growth is in the vertical axis and p_A in the horizontal. The U-shaped thick line represents expected growth with private information, $E_{z,A}[\psi(z, A)] \mid \text{private information}$. At $p_G = p_B = 0.5$, the variance of the idiosyncratic shock is maximum and the difference between private and full information expected growth is the largest. Then, we let the conditional mean of the individual shock depend on the aggregate state A , $\mu(A)$. As $p_G \leq 0.5 \leq p_B$, the discrepancy between the individual probability across aggregate states widens, the variance of the idiosyncratic shock decreases, and expected growth under private information increases monotonically to the full information value. The main lessons from Fig. 2 are: (i) the mitigating effect of aggregate uncertainty is U-shaped in p_A and decreasing in the variance of p_A ; (ii) the mitigating effect is never strong enough to reverse the inequality in Eq. (32a), that private information decreases expected growth. Thus, for either a positive or negative correlation between z and A , the effect of aggregate shocks on p can mitigate, but not reverse, the inefficiency caused by private information on expected growth.

The numerical values in the figure come from Table 2A–C, where we present numerical simulations for the case of aggregate and idiosyncratic risk with and without private information. In Table 2A, we have the case of maximum variance, $p_G = p_B = 0.5$, of the idiosyncratic shock and the discrepancy between expected growth is largest as illustrated in Fig. 2:

$$E_A[\psi(A)] \mid \text{full information} = 1.283 > E_A[\psi(A)] \mid \text{private information} = 1.281.$$

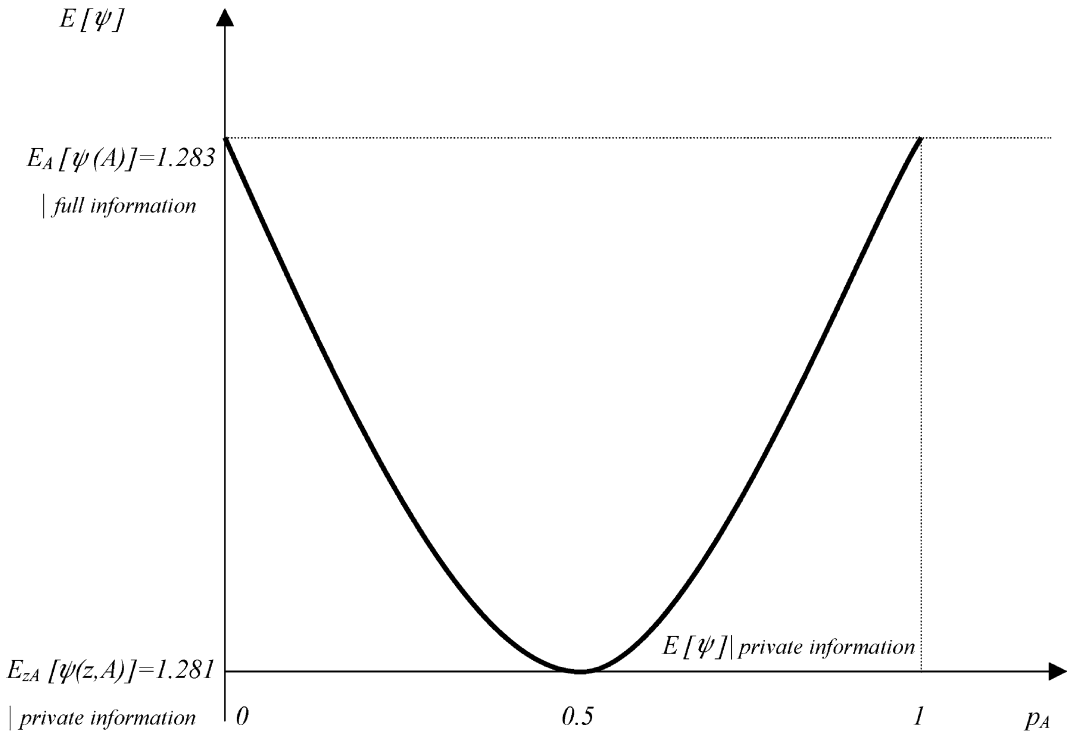


Fig. 2. Mitigating effect of aggregate uncertainty on expected growth.

In Table 2B, we decrease the variance to $p_B=0.2 < p_G=0.8$, and the gap between the expected growth decreases to

$$E_A[\psi(A)] \mid \text{full information} = 1.283 > E_A[\psi(A)] \mid \text{private information} = 1.282.$$

In Table 2C, we decrease the variance to $p_B=0.8 > p_G=0.2$, and the expected values are symmetric (U-shaped).

Therefore, providing public insurance mechanisms for aggregate shocks in the manner analyzed by Attanasio and Rios-Rull (1999) would be detrimental to expected growth under private information. Insurance to aggregate shocks would counter the mitigating effect of aggregate risk thus leaving agents bearing the negative effect of private information on expected growth.

Comparing expected growth across columns shows that aggregate uncertainty induces more substantive growth effects relative to idiosyncratic uncertainty only (a comparison of the growth factor down each column). For example, in the case of full information in Table 2A, for $p_A=0.5$, comparing columns 1 and 2, we note a change in the growth factor from the bad aggregate state to the good aggregate state of about 18 percentage points, 1.338–1.188. Examining columns 1 and 2 separately in the case of private information yields a change in the growth factor across idiosyncratic shocks of approximately 13 percentage points at most, 1.442–1.310. Therefore, the growth effects due to idiosyncratic shocks only may be “small,”

Table 2

Simulations: aggregate and idiosyncratic risk with and without private information

(A)				
$\beta=0.95$	(1) $p_A=0.5, G=1.45,$ $g=0.2, b=-0.2, \pi=0.5$	(2) $p_A=0.5, B=1.25$ $g=0.2, b=-0.2, \pi=0.5$		
<i>Private information</i>				
$c(b,A)$	0.0719	0.0620	$E_{z,A}[c(z,A)]$	0.0689
$c(g,A)$	0.0759	0.0656		
$\lambda(A)$	0.0348	0.0320		
$\eta(A)$	1.449	1.249		
$\psi(b,A)$	1.310	1.125	$E_{z,A}[\psi(z,A)]$	1.281
$\psi(g,A)$	1.442	1.247		
$\tau(b,A)$	-1.178	-0.9880	$E_{z,A}[\tau(z,A)]$	-1.281
$\tau(g,A)$	-1.574	-1.384		
ρ	0.7456	0.7456		
<i>Full information</i>				
$c(A)$	0.0725	0.0625	$E_A[c(A)]$	0.0675
$\eta(A)$	1.450	1.250		
$\psi(A)$	1.378	1.188	$E_A[\psi(A)]$	1.283
$\tau(b,A)$	-1.178	-0.9875	$E_{z,A}[\tau(z,A)]$	-1.283
$\tau(g,A)$	-1.578	-1.388		
ρ	0.7448	0.7448		
(B)				
$\beta=0.95$	(3) $p_B=0.2 < p_G=0.8, G=1.45$ $g=0.2, b=-0.2, \pi=0.5$	(4) $p_B=0.2 < p_G=0.8, B=1.25$ $g=0.2, b=-0.2, \pi=0.5$		
<i>Private information</i>				
$c(b,A)$	0.0775	0.0563	$E_{z,A}[c(z,A)]$	0.0684
$c(g,A)$	0.0800	0.0614		
$\lambda(A)$	0.0230	0.0196		
$\eta(A)$	1.569	1.129		
$\psi(b,A)$	1.382	1.049	$E_{z,A}[\psi(z,A)]$	1.282
$\psi(g,A)$	1.518	1.167		
$\tau(b,A)$	-1.172	-0.9937	$E_{z,A}[\tau(z,A)]$	-1.282
$\tau(g,A)$	-1.570	-1.389		
ρ	0.7615	0.7615		
<i>Full information</i>				
$c(A)$	0.0785	0.0565	$E_A[c(A)]$	0.0675
$\eta(A)$	1.570	1.130		
$\psi(A)$	1.492	1.074	$E_A[\psi(A)]$	1.283
$\tau(b,A)$	-1.172	-0.9935	$E_{z,A}[\tau(z,A)]$	-1.283
$\tau(g,A)$	-1.572	-1.394		
ρ	0.7609	0.7609		

Table 2 (continued)

(C)				
$\beta=0.95$	(5) $p_B=0.8 > p_G=0.2$, $G=1.45$ $g=0.2$, $b=-0.2$, $\pi=0.5$	(6) $p_B=0.8 > p_G=0.2$, $B=1.25$ $g=0.2$, $b=-0.2$, $\pi=0.5$		
<i>Private information</i>				
$c(b,A)$	0.0663	0.0677	$E_{z,A}[c(z,A)]$	0.0684
$c(g,A)$	0.0718	0.0698		
$\lambda(A)$	0.0216	0.0213		
$\eta(A)$	1.329	1.369		
$\psi(b,A)$	1.237	1.200	$E_{z,A}[\psi(z,A)]$	1.282
$\psi(g,A)$	1.365	1.326		
$\tau(b,A)$	-1.184	-0.9823	$E_{z,A}[\tau(z,A)]$	-1.282
$\tau(g,A)$	-1.578	-1.380		
ρ	0.7414	0.7414		
<i>Full information</i>				
$c(A)$	0.0665	0.0685	$E_A[c(A)]$	0.0675
$\eta(A)$	1.330	1.370		
$\psi(A)$	1.264	1.302	$E_A[\psi(A)]$	1.283
$\tau(b,A)$	-1.184	-0.9815	$E_{z,A}[\tau(z,A)]$	-1.283
$\tau(g,A)$	-1.584	-1.382		
ρ	0.7409	0.7409		

as pointed out by Khan and Ravikumar (1997), but adding aggregate uncertainty has the potential to make the growth effects of private information larger.

Comparing Table 2A–C regarding consumption behavior, the variability of individual consumption is slightly larger when $p_G=0.8 > p_B=0.2$ and smaller when $p_G=0.2 < p_B=0.8$. This confirms that aggregate uncertainty may or may not mitigate the variability of individual quantities. Comparing columns across tables for the case of private information, the variability of individual consumption is larger in the good aggregate state relative to the bad aggregate state. As a consequence, across all tables $\lambda(G) > \lambda(B)$ and the contract is more efficient in the good aggregate state since there is more variability in that state. In this case, insurance to aggregate shocks can decrease the variability of consumption when $p_G > p_B$ thus making private insurance less efficient.¹⁵

Finally, we look at the row for the discount factor, ρ . First, notice that when $p_G=0.8 > p_B=0.2$ (Table 2B), the discount factor increases so that the risk-free interest rate decreases improving the excess return. However, when $p_G=0.2 < p_B=0.8$ (Table 2C), the discount factor decreases so that the risk-free interest rate increases thus reducing the excess return, again confirming that aggregate uncertainty may affect the excess return both ways. In all cases, the inequality in Eq. (32b) is preserved so that aggregate uncertainty does not reverse the result that private information increases the excess return relative to full information.

¹⁵ Attanasio and Rios-Rull (1999) show that public insurance can distort (crowd out) private insurance mechanisms and decrease welfare of individuals. Our result above regards the efficiency of the contract in terms of the marginal efficiency of the contract, $\lambda(A)$.

5. Conclusions

We argue that aggregate uncertainty is potentially important for the individual decision-making process. Idiosyncratic uncertainty alone seems to yield a plausible explanation for the discrepancy in the variability of individual versus aggregate per individual quantities. Adding aggregate uncertainty provides possible additional channels that can either increase or decrease the variability of individual versus aggregate per individual quantities. The end result is sensitive to the way aggregate uncertainty affects the probability distribution of the idiosyncratic shock, i.e., the sign of the correlation between individual and aggregate risk. We show that cases where the individual variability may decrease are associated with a probability of high individual productivity being large when the aggregate shock is bad, i.e., the correlation between individual and aggregate risk is negative. We confirm the result of Khan and Ravikumar (1997) who found that idiosyncratic uncertainty under private information decreases expected growth. We show that aggregate uncertainty can mitigate the effect of idiosyncratic uncertainty but cannot reverse those results. We basically show that effects of private information are sensitive to whether or not aggregate uncertainty is taken fully into account and whether or not aggregate uncertainty affects the probability distribution of idiosyncratic shocks. Thus, aggregate shocks and individual private information may have larger growth effects. Moreover, insurance mechanisms against aggregate shocks would be detrimental to expected growth in the presence of private information.

The effects of aggregate and idiosyncratic uncertainty on the risk-free asset price were also examined and they work in the plausible direction of increasing the excess return by decreasing the risk-free return (e.g., Heaton & Lucas, 1992). However, we show that this result is also sensitive to the sign of the correlation between individual and aggregate risk.

Further research regarding extensions to the more general isoelastic utility function and issues relating to income distribution is certainly worth pursuing. A more important avenue regards the foundations of the relationship between aggregate shocks and the probability distribution of individual idiosyncratic shocks.

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