A dynamic monetary model with costly foreign currency

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Abstract

I present a dynamic general equilibrium monetary model with domestic and foreign currencies and a traded bond where there is an adjustment cost to switch into foreign currency. The focus is on the short versus long run trade-offs and transitional dynamics of domestic and foreign monetary disturbances as a function of attributes of currencies in utility. The main finding is that short and long run trade-offs and transitional dynamics together with the implied hysteresis property of the equilibrium are critical determinants of the qualitative results of domestic and foreign monetary disturbances in this class of model.

Keywords

Monetary policy, foreign inflation, hysteresis, adjustment cost, currency substitution

1. INTRODUCTION

The effects of domestic monetary policy and the foreign transmission of inflation in a world of flexible exchange rates with high capital mobility have become a topic of importance, mainly since the unambiguous move to a system of flexible rates and the continuous integration of capital markets in the last 20 years. Even though these changes have been evident, the preservation of national currencies along with national monetary policy autonomy has yet to be seriously challenged. Here, I present a dynamic model that examines the issues of domestic monetary policy and the transmission of foreign inflation when the representative individual has a menu of assets that includes a foreign currency.

The model is a dynamic general equilibrium one that solves for the allocation of an individual's wealth into domestic and foreign currencies and a domestically traded bond when it is costly to switch into the foreign currency. These transactions costs emerge from frictions involved in acquiring foreign currency in the market place. Even though technological advances in the
financial industry sector and financial liberalization have made these costs less restrictive more recently (see for example Guidotti and Rodriguez, 1992; Guidotti, 1993; Eaton, 1994) their importance cannot be ignored. Authors such as Bana and Handa (1990) have included a variable reflecting these costs in a framework of currency substitution and found it to be statistically significant. Others, such as Black (1991), have examined how vehicle currencies emerge in an attempt to diminish these transactions costs. Specifically, domestic and foreign currencies are assumed to provide liquidity services by entering directly in the utility function. This approach has been extensively used in the general macroeconomic monetary framework since Sidrausky (1967). In a multiple currency framework, it follows the tradition of Calvo (1985), Engel (1989), Rogers (1990), Weil (1991), Aizenman (1992), Canzoneri and Diba (1992, 1993), Imrohoroglu (1996), and Uribe (1997).

In particular, this paper explores the dichotomy of currencies being imperfect substitutes/imperfect complements versus competitive in utility. The model with traded bonds and with the cost of switching into foreign currency is a particularly useful extension of Engel (1989) because, contrary to his results, it does present transitional dynamic adjustment. In addition, it presents the hysteresis property that the derived equilibrium depends on initial conditions according to the unit root hypothesis in time series data. In this paper, the hysteresis result comes from a portfolio balance effect as follows. A temporary change in expected inflation permanently affects the initial stock of domestically traded bonds, therefore changing the mixture of bonds, domestic, and foreign currency in the individual portfolio. Therefore, temporary changes in expected inflation permanently affect the degree of currency substitution. In the recent paper by Uribe (1997), the hysteresis result is generated by a network externality in which the transactions costs of using foreign currency are inversely related to the domestic agent's accumulated experience in using it. In that paper, temporary changes in expected inflation may permanently affect the velocity of circulation and therefore the degree of currency substitution. Alternatively, my paper focuses on the portfolio balance effect. I also show the general source-based property of the inflation tax, hence the implied effects of domestic holdings of foreign currency on the current account balance.

My study highlights two basic issues: (i) the transitional dynamic paths as a function of the (imperfect) complements/substitute and competitive aspects of domestic and foreign currencies in the utility function and as a function of the relative magnitudes of the foreign inflation rate versus the speed of adjustment in domestic asset markets; (ii) the short versus long run trade-offs and transitional dynamics of domestic and foreign monetary disturbances also as a function of the (imperfect) complement/substitute and competitive aspect of currencies in the utility function. The main lesson to be learned is that short and long run trade-offs and transitional dynamics together with the
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hysteresis property of the equilibrium are critical determinants of the qualitative results of domestic and foreign monetary disturbances in this class of model.

The paper is organized as follows. Section 2 outlines the model, gives its solution, and derives the dynamic adjustment paths. Section 3 presents the effects of domestic and foreign monetary disturbances, while Section 4 concludes. An Appendix presents details of the solution of the model.

2. MACROECONOMIC STRUCTURE

Consider a continuous-time perfect-foresight one-good model of a decentralized small open economy inhabited by households and a government. This is a pure endowment economy, abstracting from issues of production. With one good, the law of one price holds such that \( P = E P^* \), where \( P \) is the price of the domestic good, \( P^* \) is the price of the foreign good and \( E \) is the nominal exchange rate as units of domestic currency per unit of foreign currency. Logarithmic time derivatives of the law of one price imply that the rate of change of the nominal exchange rate obeys

\[
\frac{dE}{dt} = n - x^*,
\]

where \( n = \frac{dP}{dt} / P \) is the domestic inflation rate, and \( n^* = \frac{dP^*}{dt} / P^* \) is the foreign inflation rate assumed to be exogenously given.

An individual in this economy can hold three assets: domestic money, foreign money, and traded bonds. There is perfect capital mobility, and foreign money and bonds are assumed to be available to the individual at a perfectly elastic supply. The feature of the model is that the individual faces an adjustment cost for switching into foreign currency. Let the change in nominal foreign money balances divided by the foreign price level be defined as \( \Phi^* \), and given by

\[
\Phi^* = \frac{\dot{M}^*}{P^*} = \dot{m}^* + \pi^* m^*
\]

where \( m^* \) is the stock of foreign currency, in real terms, held by the domestic resident. From the expression above, accumulating \( m^* \) incurs an adjustment cost of the form \( \phi(\Phi^* - \pi^* m^*) \) with the function \( \phi(.) \) satisfying the following:

\[ \phi'(.) > 0; \quad \phi''(.) > 0; \quad \phi(0) = 0; \quad \text{and} \quad \phi'(0) = 1. \]

This convex cost reflects the resources spent on switching from domestic money or traded bonds into foreign currency. I assume it takes the specific functional form

\[
\phi(\Phi^* - \pi^* m^*) = (\Phi^* - \pi^* m^*) \left[ 1 + \left( \frac{1}{2} \right) h (\Phi^* - \pi^* m^*) \right]
\]

for \( h \in (0, \infty) \). Domestic and foreign currencies provide liquidity services by entering directly in the utility function as in Calvo (1985), Engel (1989), Rogers (1990), Weil (1991), Aizenman (1992), Canzoneri and Diba (1992) and Imrohoroglu (1996) among others. I proceed by presenting the model for the households, government, and the wealth and accumulation of foreign assets accounting.
2.1 Households

The representative household in the domestic country faces the intertemporal problem

$$\begin{align*}
\text{Max} & \int_0^\infty \left[ U(c) + V(m, m^*) \right] (\exp - \delta t) \, dt \\
\text{subject to} & \quad c + \dot{m} + \phi(\Phi^* - \pi^* m^*) + \dot{b}^* = y_c + r^* b^* - \pi m - \pi^* m^* + T
\end{align*}$$

(2)

subject to the budget constraint,

$$c + \dot{m} + \phi(\Phi^* - \pi^* m^*) + \dot{b}^* = y_c + r^* b^* - \pi m - \pi^* m^* + T$$

(2a)

the equation for the accumulation of foreign real money balances, or net investment in foreign currency,

$$\dot{m}^* = \Phi^* - \pi^* m^*$$

(2b)

and given initial holdings

$$m_0 > 0, \quad m^*_0 > 0, \quad b^*_0$$

(2c)

where $c$ is real consumption, $m$ is the real stock of domestic currency held by the domestic resident, $\delta \in [0, \infty)$ is the individual rate of time preference taken as given, $b^*$ is the real stock of traded bonds owned by domestic residents (denominated in foreign currency), $y_c \in [0, \infty)$ is the endowment of the individual, $r^* \in [0, \infty)$ is the real return on traded bonds taken as given by the domestic resident, and $T$ is a lump-sum real transfer.

The instantaneous utility function comprises two subfunctions, $U(.)$ and $V(., .)$, for which I assume the following functional forms:

$$U(c) = \log(c)$$

(2d)

$$V(m, m^*) = (m^\beta + m^*^\beta)^{(\alpha/\beta)}$$

(2e)

with $\alpha \in (0, 1]$ and $\beta \in (-\infty, 1]$. For the specification (2e), the elasticity of substitution between the two currencies is $1/(1 - \beta)$. The sign of the cross partial $V_{12} = V_{21}$, depends on the (imperfect) substitutability/complementarity versus competitiveness of the currencies in utility through the parameters $\alpha$ and $\beta$:

$$V_{12} = V_{21} = \alpha (\alpha - \beta) (m m^*)^{(\beta - 1)} (m^\beta + m^*^\beta)^{(\alpha/\beta)} V(m, m^*)$$

(2f)

If the two currencies are assumed to be imperfect complements/imperfect substitutes in utility, as in Calvo (1985), then $V_{12} = V_{21} > 0$ and the parameter space is $\beta < \alpha \leq 1$. In this case, as individuals increase their holdings of one currency, it increases the marginal utility of the other currency, but currencies may be imperfect substitutes, thus showing some degree of complementarity. Alternatively, if they are assumed to be competitive in utility, as in Engel (1989), then $V_{12} = V_{21} < 0$ and the parameter space is $\alpha < \beta \leq 1$, and the two currencies function as competitive factors in utility. If only the sum of the
two moneys matter for utility, then the moneys become perfect substitutes. There are different combinations of the parameters \( \alpha \) and \( \beta \) that yield this case. For example, in the polar case where \( \alpha = \beta = 1 \), the elasticity of substitution between the two currencies approaches infinity and the two currencies become perfect substitutes in utility, \( V(m, m^*) \rightarrow (m + m^*) \) and \( V_{12} \rightarrow 0 \) in this case.\(^6\) In the other extreme, for any given \( \alpha \), as \( \beta \rightarrow -\infty \) the elasticity of substitution between the two currencies approaches zero and \( V_{12} \rightarrow \infty \).

The household problem is standard. The budget constraint is expressed in real flow terms and consists of the endowment plus returns from holding traded bonds less the inflation tax on holdings of domestic and foreign currencies plus the lump-sum transfer (right-hand side). This is to be spent on consumption or additions to the stock of domestic and/or foreign money and bonds (left-hand side), where the accumulation of foreign real money balances is subject to the transaction cost \( \phi(\cdot) \).\(^7\)

The first-order necessary conditions to this problem are

\[
U'(c) = \lambda \tag{3a}
\]

\[
V_i(m, m^*) - \pi = \delta - (\lambda/\lambda) \tag{3b}
\]

\[
\phi'(\Phi^* - \pi^* m^*) = (\mu/\lambda) \tag{3c}
\]

\[
[V_2(m, m^*)/\mu] + \pi^*[\phi'(\Phi^* - \pi^* m^*) - 1] (\lambda/\mu) - \pi^* = \delta - (\mu/\mu) \tag{3d}
\]

\[
r^* = \delta - (\lambda/\lambda) \tag{3e}
\]

together with the transversality conditions

\[
\lim_{t \rightarrow \infty} \lambda m(\exp -\delta t) = 0; \quad \lim_{t \rightarrow \infty} (\lambda + \mu) m^* (\exp -\delta t) = 0; \quad \lim_{t \rightarrow \infty} \lambda b^*(\exp -\delta t) = 0 \tag{3f}
\]

where \( \lambda \), the non-negative Lagrange multiplier associated with the accumulation equation (2a), is the marginal utility of wealth of the domestic resident, and \( \mu \), the non-negative Lagrange multiplier associated with the accumulation of foreign money equation (2b), is the marginal utility of net investment in foreign currency. Equations (3b, d) are arbitrage conditions stating that the domestic consumer must be indifferent between the rates of return on domestic money, foreign money and consumption. Equation (3c) equates the marginal cost of investing in foreign currency to its marginal benefit. Equation (3e) is an arbitrage condition equating the return on foreign bonds to the return on consumption. Since there is perfect capital mobility and \( r^* \) and \( \delta \) are constants, it is well known in this class of model that the assumption that \( r^* = \delta \) is required. This implies perfect consumption smoothing, or

\[
(\lambda/\lambda) = 0 \quad \text{and} \quad \lambda = \lambda_c \text{ for all } t > 0 \tag{4}
\]
2.2 Government

The government budget is balanced according to

$$\dot{m} = T - \pi m$$  \hspace{1cm} (5)

which says that the deficit emerging from real monetary transfers minus the inflation tax is financed by the issuance of currency. The domestic monetary policy is specified according to Friedman's constant rate of growth of nominal money balances, $\sigma = (dM/dt)/M > 0$, such that

$$\dot{m} = (\sigma - \pi) m$$  \hspace{1cm} (6)

The budget constraint (5) and monetary policy (6) imply the usual condition that lump-sum real transfers adjust to balance the budget according to $T = \sigma m$; see for example Engel (1989).

2.3 Wealth and accumulation of foreign assets

Aggregate domestic real wealth is defined as

$$W = m + m^* + b^*$$  \hspace{1cm} (7)

where $m$ and $m^*$ denote the stocks of domestic and foreign real money balances owned by domestic residents, and $b^*$ the real stock of traded bonds owned by domestic residents. It is the permanent effect of temporary changes in expected inflation that will have a permanent effect on the allocation of wealth among these three assets, thus leading to the hysteresis result.

The net foreign asset position of the domestic economy is simply $b^*$, the stock of traded bonds. After some algebraic manipulation using the time derivative of $b^*$, the household budget constraint (2a), the government budget constraint (5), and the government monetary policy (6), one obtains an expression for the rate of accumulation of foreign assets, or the current account balance, as

$$\dot{b}^* = (y_c - c) + r^* b^* - [\phi(\Phi^* - \pi^* m^*) + \pi^* m^*]$$  \hspace{1cm} (8)

given initial holdings, $b^*_0$. This expression simply states that the change in foreign assets is the trade balance, $(y_c - c)$, plus the flow of earnings from the stock of foreign assets, $r^* b^*$, minus a term reflecting the inflation tax that accrues to the foreign government.\(^8\)

Equation (8) captures the effects of the inflation tax on the current account; see for example Fischer (1982) and Guidotti (1993). Usually in this class of models, domestic residents do not hold foreign currency and the term $-[\phi(\Phi^* - \pi^* m^*) + \pi^* m^*]$ does not appear in the current account equation. This effect is founded on the incidence property of the inflation tax: the inflation tax is always a source-based tax, that is, it is paid by whoever holds the currency independently of the country in which the individual is located.\(^9\)
This quantity is of importance both in theory and practice. Here, in the theory, it turns out to be of crucial importance to determine the dynamic adjustment between traded bonds and domestic and foreign currency along the transitional path. In practice, it is quite possible that residents of certain countries hold an amount of foreign currency that could impact on the current account, the well-known 'capital flight' problem, see for example Dooley and Kletzer (1994) and Cumby and Levich (1988).

2.4 Equilibrium and dynamics

The short-run equilibrium for consumption, domestic inflation, and gross investment in foreign currency is given from (3a, b, c) by

\[c_c = c(\lambda_c)\]
\[c_x = (1/U'') < 0\]
\[\pi = \pi(m, m^*); \quad \pi_1 = V_{11}/\lambda_c < 0, \quad \pi_2 = V_{12}/\lambda_c\]
\[\Phi^* = \Phi^*(\mu, m^*); \quad \Phi^*_{11} = (1/\lambda_c \phi'') > 0, \quad \Phi^*_{22} = \pi^* > 0\]

Using these short-run relationships and equations (3b), (4) and (6), the dynamic evolution of the economy may be represented by

\[\dot{m} = m \{(\delta + \sigma) - [V_1(m, m^*)/\lambda_c]\}\]
\[\dot{m}^* = \Phi^*(\mu, m^*) - \pi^* m^*\]
\[\dot{\mu} = \mu(\delta + \pi^*) - V_2(m, m^*) - \pi^*(\mu - \lambda_c)\]

Together with the evolution of foreign assets given by (8), which is subject to the usual condition that prevents the economy from running up infinite debt, that is

\[\lim_{t \to \infty} b^*[\exp \int_0^t r^*(s) \, ds] = 0\]

The dynamic adjustment to the steady-state equilibrium is analysed by the usual method of linearizing the dynamical system around a steady-state equilibrium. Equations (10a, b, c) represent a differential system in the three variables \(m, m^*\) and \(\mu\). Linearizing around the steady-state equilibrium, \(m_c, m^*_c\) and \(\mu_c\), yields a 3 \times 3 matrix whose determinant is negative and trace is positive as described in the Appendix (equation (A1)). This implies that one of the three characteristic roots that govern the dynamic adjustment is negative while the other two are positive; that is, saddlepoint dynamics. Denoting by \(\gamma < 0\) the stable root of the characteristic equation, the dynamic adjustment of \(m, m^*\) and \(\mu\), for every time \(t\), is described by

\[m(t) = m_c + [m(0) - m_c] (\exp \gamma t)\]
\[m^*(t) = m^*_c + \Psi^* [m(0) - m_c] (\exp \gamma t)\]
The solution for the foreign asset position, $b^*$, is recovered by applying a procedure discussed by Sen and Turnovsky (1990) (see the Appendix, equations (A2)-(A3)). This yields a path for $b^*$ described by

$$b^*(t) = b^* + [Q - \psi Y - \phi (m(0) - m_1)] \exp \gamma t$$

where $Q = -\Phi \Psi - \Phi \Psi$. The system (12a-d) is the solution for the dynamics of domestic real money balances, foreign real money balances, the shadow value of foreign real money balances and the holdings of traded bonds along the equilibrium dynamic path. The steady state equilibrium is described in the Appendix (equations (A4)-(A12)).

The equilibrium derived in my model is very different from the equilibrium in Rogers (1990) for two basic reasons. First, Rogers (1990) has an endogenous rate of time preference whereas my model has a fixed rate of time preference with an adjustment cost to foreign currency holdings. Second, my model has the property that temporary disturbances have permanent effects since the stationary state derived in my framework depends on initial conditions, whereas Rogers (1990) does not share this property. This hysteresis result is shared by, for example, Sen and Turnovsky (1990) and Uribe (1997). This is a desirable property of my model since actual time series data, such as domestic real money balances and foreign denominated asset holdings, do present unit roots. As I show below, these are important differences that lead to fundamentally different results from Rogers (1990) regarding foreign inflation transmission, as well as from the results of Imrohoroglu (1996) who only analyses the stationary state effects of domestic monetary growth disturbances without the short versus long run trade-offs and transitional dynamics obtained here.

### 2.5 Domestic and foreign currencies and the net foreign asset position adjustment

**The $m - m^*$ locus**

From the equilibrium dynamics (12a, b), the stable dynamic path between the demands for domestic and foreign currency is given by

$$m(t) - m_c = [- (m_c V_{12})/(\gamma \lambda_c + m_c V_{11})] [m^*(t) - m^*]$$

The denominator of the slope of the stable manifold is unambiguously negative while the numerator depends on the sign of $V_{12}$ or the parameters $\alpha$ and $\beta$ in the subutility function $V(.,.,.)$. First, if $V_{12} > 0$ or $\beta < \alpha \leq 1$, that is the two currencies are imperfect complement/imperfect substitutes in utility as in Calvo (1985), then the slope of the manifold is positive, the accumulation or
decumulation of the two currencies are positively related, and more of one currency helps the marginal utility of the other currency. Alternatively, if $V_{12} < 0$ or $\alpha < \beta \leq 1$ as in Engel (1989), the two currencies are competitive in utility and the slope of the manifold is negative. As individuals increase the holdings of one currency, it decreases the marginal utility of the other currency.\(^1\)

**The $m - b^*$ locus**

From equations (12a, d), the stable dynamic path between domestic real money balances and the foreign asset position is described by

$$m(t) - m_c = \{(\gamma - \delta)/(\gamma \alpha_c + m_c V_{11})((\gamma + \pi^*)/(m_c V_{12}))\} [b^*(t) - b^*_c]$$

(14)

The numerator of the slope is unambiguously negative whereas the denominator depends on the sign of two parameters: $V_{12}$ and $\gamma + \pi^*$. The effect of the sign of $\gamma + \pi^*$ on the $m - b^*$ locus derives from the foreign inflation tax effect in equation (8). For a given sign of $V_{12}$, the magnitude and sign of $\gamma + \pi^*$ regulates the flow of foreign assets relative to the flow of foreign currency through the inflation tax channel in equation (8), the term $-\pi^* m^*$. The various possible combinations are as follows.

(i) $V_{12} > 0$ and $\gamma < -\pi^*$; the currencies are imperfect complement/imperfect substitutes in utility and the foreign rate of inflation is greater than the speed of adjustment in the asset markets (or smaller in absolute value); the slope of the manifold is negative; that is, as the holdings of domestic currency increase, the foreign asset position deteriorates and the economy suffers a current account deficit.

(ii) $V_{12} > 0$ and $\gamma > -\pi^*$; the currencies are imperfect complement/substitutes in utility and the foreign rate of inflation is smaller than the speed of adjustment in the asset markets (or greater in absolute value); the slope here is positive; that is, the more holdings of domestic currency the more foreign assets the individual accumulates with a consequent current account surplus.

(iii) $V_{12} < 0$ and $\gamma < -\pi^*$; the two currencies are competitive in utility and the foreign rate of inflation is greater than the speed of adjustment in the asset markets (or smaller in absolute value); the slope of the locus is positive.

(iv) $V_{12} < 0$ and $\gamma > -\pi^*$; the two currencies are competitive in utility and the foreign rate of inflation is smaller than the speed of adjustment in the asset markets (or greater in absolute value); the slope of the locus is negative.\(^12\)

As mentioned above, the relationship between the sign of $\gamma + \pi^*$ and the $m - b^*$ locus derives from the foreign inflation tax effect in equation (8). For example, if $V_{12} > 0$ and $\gamma < -\pi^*$ as in case (i), then the inflation tax incurred in holdings of foreign currency is greater than the rate at which the individual accumulates assets. This foreign inflation tax effect dominates the offsetting effect from the speed of adjustment, and $m$ and $b^*$ are negatively related along the transitional
path. Alternatively, if $V_{12} > 0$ and $\gamma > -\pi^*$ as in case (ii), the speed of adjustment effect dominates, and $m$ and $b^*$ are positively related along the transitional path. Lastly, in the polar cases of $\beta \to 1$ and $V_{12} \to \infty$ the $m - b^*$ locus becomes respectively horizontal and vertical as in the case of the $m - m^*$ locus. The path may also be vertical if $\gamma \to -\pi^*$, or the speed of adjustment in asset markets in the domestic economy is equal to the foreign rate of inflation.

3. DYNAMICS OF DOMESTIC AND FOREIGN MONETARY DISTURBANCES

The comparative dynamics of monetary disturbances are obtained directly from the short run equilibrium conditions (9a)-(9c), the dynamic paths in (12a)-(12d), and the steady state equilibrium in (A4)-(A12). Since the individual takes the foreign price level as given and that to switch into foreign currency is initially costly, I assume that when the system is subject to a certain disturbance, the stock of foreign real money balances, $m^*$, and the stock of traded bonds, $b^*$, are predetermined, hence $dm^*_0 = db^*_0 = 0$. In turn, it is the marginal utility of net investment in foreign currency $p(0)$ that responds instantaneously together with the domestic price level. Thus, the initial stock of wealth of the domestic resident is going to change according to the wealth effect determined by an instantaneous change in the domestic price level. When the economy is subject to a disturbance there is an instantaneous change in the domestic price level that implies an adjustment in domestic real money balances with \[^{14}\]

$$dm(0) = dW(0) \quad (15)$$

A natural measure of welfare of the representative agent is the discounted utility in equation (2). I compute the welfare effects of the disturbances by taking a linear approximation of the functional in (2), substituting (9a) and (12b) and directly evaluating the integral, see for example Turnovsky (1992). The resulting change in discounted utility denoted by $Z$ is given by

$$dZ = \lambda_c dc_c + V_1 dm_c + V_2 dm^*_c + [(V_1 + V_2 \Psi^*)/(\delta - \gamma)] [dm(0) - dm_c] \quad (16)$$

which can be directly computed for each of the alternative disturbances.

3.1 Domestic monetary growth disturbances, $d\sigma$

The solution for an increase in the domestic monetary growth rate is obtained by substituting (12b) and (A12) using (15) into (A4)-(A7) yielding the effects on $m_c, m^*_c$ and $c_c$ (presented in the Appendix, (A13)-(A20)). The subsequent effects can be obtained recursively. A summary of the results in $m, m^*$ and $b^*$ space is shown in Figure 1.

The initial equilibrium is at point A. An increase in $\sigma$, $d\sigma > 0$, leads to two
Figure 1 Domestic monetary growth disturbance, \( d\sigma > 0 \)

different transitional dynamic paths depending on the sign of \( V_{12} \). In the upper panel (a) \( V_{12} > 0 \). The initial stock of domestic real money balances falls instantaneously because there is an increase in the cost of holding domestic currency which leads to an instantaneous increase in the domestic price level. This is the jump from points A to A'. Since the two currencies are imperfect complements in utility, \( V_{12} > 0 \), both stocks of real money balances decrease
The stock of foreign bonds also gradually falls, indicating a current account deficit in the transition. The effects are summarized as follows:

\[ \frac{dm(0)}{d\sigma_{V_{12}}} > 0 < 0 \]  

(17a)

\[ \frac{d m_c}{d \sigma_{V_{12}}} > 0; \quad \frac{d m^*}{d \sigma_{V_{12}}} > 0; \quad \frac{dc_c}{d \sigma_{V_{12}}} > 0; \quad \frac{db^*}{d \sigma_{V_{12}}} > 0 \]  

(17b)

\[ \frac{d Z}{d \sigma_{V_{12}}} > 0 \]  

(17c)

Private consumption decreases once-and-for-all while the welfare of the representative agent increases because of the transitional effect. It is important to highlight the implied mechanism here. An increase in the domestic monetary growth rate decreases steady-state values of consumption, traded bonds, and foreign money. In the current account equation, (8), all these effects are counteracted by a gradual decrease in the holdings of traded bonds, which decreases the interest receipts on holdings of these bonds. The outcome of these two opposing effects is in favour of the former, which implies a current account deficit along the transitional path.

The lower panel (b) of Figure 1 presents the case \( V_{12} < 0 \). The instantaneous price level effect is similar, but the transition is different. With \( V_{12} < 0 \), or competitiveness among the currencies in utility, along the transition path, the stock of domestic real money balances increases gradually while the stock of foreign real money balances decreases to point B. There is also a current account deficit along the transition and the long run effect on the stock of traded bonds is negative. The effects are summarized as follows:

\[ \frac{dm(0)}{d\sigma_{V_{12}}} < 0 \]  

(18a)

\[ \frac{d m_c}{d \sigma_{V_{12}}} < 0; \quad \frac{d m^*}{d \sigma_{V_{12}}} < 0; \quad \frac{dc_c}{d \sigma_{V_{12}}} < 0; \quad \frac{db^*}{d \sigma_{V_{12}}} < 0 \]  

(18b)

\[ \frac{d Z}{d \sigma_{V_{12}}} < 0 \]  

(18c)

Again, private consumption decreases once-and-for-all and the welfare effect is negative in this case.

These results highlight the importance of the sign of \( V_{12} \); the transitional dynamics and the short versus long run trade-offs that domestic monetary disturbances imply in my model. First, when \( V_{12} > 0 \) an increase in the domestic monetary growth rate triggers the usual inflation tax effect on domestic real money balances in the short and long run. However, because the currencies are (imperfect) complements in utility, along the transition, as holdings of foreign real money balances decrease, so do holdings of domestic real money balances. In the case where \( V_{12} < 0 \), the initial impact of the inflation tax effect is similar, but along the transition individuals accumulate
domestic money while holding less foreign money. The steady-state effect is
an increase in domestic money holdings.

This positive relation in the steady state between domestic money growth
and domestic real money balances in the case where $V_{12} < 0$ implies that the
inflation tax may not be subject to Laffer-style effects, a result also pointed out
recently by Imrohoroglu (1996). My results show that this conclusion should
be viewed with caution. Imrohoroglu (1996) only examines the stationary
state. When short run and transitional dynamics are taken into account, the
overall gain in domestic money holdings trades off with the initial inflation tax
effect, which may indeed reduce the present discounted value of future
inflation tax revenues. Define the inflation tax revenue from domestic money
holdings as

$$RV(t) = \pi(t) m(t)$$

(19a)

which implies impact and long run effects given by

$$\frac{dRV(0)}{d\sigma} = (1/\sigma) \left(\frac{dm(0)}{d\sigma}\right) \left[1 + \left(1/\epsilon_{m,\pi}\right)\right]$$

(19b)

$$\frac{dRV_c}{d\sigma} = (1/m_c) \left[1 + \epsilon_{m,\pi}\right]$$

(19c)

where $\epsilon_{m,\pi} \equiv (\sigma/m_c) \left(\frac{dm_c}{d\pi}\right) \equiv (\sigma/m_c) \left(1/\pi_c\right) \leq 0$ is the short run elasticity of
domestic money demand with respect to changes in the short run inflation rate
and $\epsilon_{m,\sigma} \equiv (\sigma/m_c) \left(\frac{dm_c}{d\sigma}\right)$ is the long run elasticity of domestic money
demand with respect to changes in the long run inflation rate determined by
the domestic rate of growth of money. Hence, except for the case where the
short and long run elasticities of money demand are unit elastic, the short and
long run effects are non-trivial. If the short run money demand is elastic
(inelastic) or

$$\epsilon_{m,\pi} < -1 \ (> -1)$$

then

$$\frac{dRV(0)}{d\sigma} < 0 \ (> 0)$$

The same result applies to the long run elasticity, plus the possibility in (18b)
discussed above. By direct computation, the change in the present discounted
value of the inflation tax revenues, $RVD$, is given by

$$\frac{dRVD}{d\sigma} = \left(\frac{dRV_c}{d\sigma}\right)/\delta + \left[\left(\frac{dRV(0)}{d\sigma}\right) - \left(\frac{dRV_c}{d\sigma}\right)\right]/(\delta - \gamma)$$

(19d)

which can be easily evaluated for both cases above yielding:

$$\frac{dRVD}{d\sigma}\big|_{V_{12} > 0} < 0; \text{ and } \frac{dRVD}{d\sigma}\big|_{V_{12} < 0} > 0$$

(19e)

Thus, in both cases the discounted value of inflation tax revenue falls, and this
is exactly the reverse of the steady-state result of Imrohoroglu (1996).15 In the
case when $V_{12} > 0$, real money balances fall both in the short and long run and
the revenue from inflation falls accordingly. When $V_{12} < 0$, it is the initial short
run fall in real money balances that drives the decrease in the discounted value of inflation tax revenue. Imrohoruglu (1996) ignores this short run effect and obtains a reversed result. It is therefore very important to consider not only the steady-state effect but also the impact effect together with the transitional dynamics. Note also that the domestic resident is unambiguously worse off in welfare terms when $V_{12} < 0$, but better off when $V_{12} > 0$. These welfare effects are indeed due to the impact versus transitional dynamic effects.

3.2 Foreign inflation disturbances, $d\pi^*$

Figure 2 illustrates a change in foreign inflation, $d\pi^* > 0$. The upper panel (a) presents the case where $V_{12} > 0$. The effects are summarized by (see Appendix, (A13)–(A20)):

$$ dm(0)/d\pi^*|_{V_{12} > 0} < 0 $$  \hspace{1cm} (20a)

$$ dm_{c}/d\pi^*|_{V_{12} > 0} < 0; \ dm_{s}/d\pi^*|_{V_{12} > 0} < 0; \ dc_{c}/d\pi^*|_{V_{12} > 0} < 0; \ db^*/d\pi^*|_{V_{12} > 0} < 0; $$  \hspace{1cm} (20b)

$$ dZ/d\pi^*|_{V_{12} > 0} > 0 $$  \hspace{1cm} (20c)

Domestic real money balances instantaneously decrease given the initial increase in the domestic price level triggered by $d\pi^*$. This is illustrated by the jump from points $A$ to $A'$. Along the transition, holdings of the three assets gradually fall with the associated current account deficit. Private consumption adjusts (instantaneously) downwards but overall welfare increases due to the transitional effects. Since there is (imperfect) complementarity among the two currencies, the comparative dynamics here are similar to the case of an increase in the rate of growth of domestic currency.

The lower panel (b) presents the case where $V_{12} < 0$. The effects are summarized by:

$$ dm(0)/d\pi^*|_{V_{12} < 0} < 0 $$  \hspace{1cm} (21a)

$$ dm_{c}/d\pi^*|_{V_{12} < 0} < 0; \ dm_{s}/d\pi^*|_{V_{12} < 0} < 0; \ dc_{c}/d\pi^*|_{V_{12} < 0} > 0; \ db^*/d\pi^*|_{V_{12} < 0} < 0 $$  \hspace{1cm} (21b)

$$ dZ/d\pi^*|_{V_{12} < 0} > 0 $$  \hspace{1cm} (21c)

The impact effect on the asset holdings is similar to the $V_{12} > 0$ case. Along the transition the results are different. Holdings of domestic real money increase, but the holdings of foreign currency and traded bonds gradually decrease. The economy runs a current account deficit partly because private consumption adjusts (instantaneously) upwards and welfare is improved due to the instantaneous and transitional effects.

These results indicate that it is only along the transitional path, when $V_{12} < 0$, that an increase in foreign inflation induces substitution into the domestic currency. In the long run there is an overall decrease of domestic real money
balances, regardless of the sign of $V_{12}$. This is contrary to the general result of Rogers (1990) where $V_{12} < 0$ is shown to be a sufficient condition for substitution into the domestic currency in the long run. The reasons for the difference are (i) the introduction of the cost of adjustment to acquire foreign currency; (ii) the initial wealth effect on domestic real money balances; and (iii) the dependence of the long run equilibrium on initial conditions, i.e. the
hysteresis property. These are properties that characterize my model relative to the alternative specification of Rogers (1990). In both cases, my results show that foreign money holdings fall in the long run; however, an analogous principle that applied to the case of an increase in the domestic monetary growth rate may apply here in terms of foreign inflation tax revenue. Computation of foreign inflation tax revenue

\[ RV^*(t) = \pi^*(t) m^*(t) \]  

(22a)
yields impact and long run effects as

\[ \frac{dRV^*(0)}{\pi^*} = m^*_c > 0 \]  

(22b)
\[ \frac{dRV^*}{\pi^*} = \frac{(1/m^*_c)}{[1 + \epsilon^*_{m, \pi}]} \]  

(22c)
where \( \epsilon^*_{m, \pi} = (\pi^*/m^*_c) (dm^*_c/\pi^*) < 0 \) is the long run elasticity of demand for foreign currency with respect to changes in the foreign inflation rate. If the demand is elastic (inelastic) or

\[ \epsilon^*_{m, \pi} < -1 \ (>-1) \]

then

\[ \frac{dRV^*}{\pi^*} < 0 \ (>0) \]

The change in the discounted value of the inflation tax revenues, \( RVD^* \), is given by

\[ dRVD^*/\pi^* = (dRV^*_c/\pi^*)/\delta + [(dRV^*(0)/\pi^*) - (dRV^*_c/\pi^*)]/(\delta - \gamma) \]  

(22d)
yielding:

\[ dRVD^*/\pi^*|_{V_{12} > 0} > 0 \; \text{and} \; dRVD^*/\pi^*|_{V_{12} < 0} > 0 \]  

(22e)
Hence, regardless of the sign of \( V_{12} \), the discounted value of foreign inflation tax revenue increases. Obviously the short run impact effect is always positive and independent of the short run elasticity of foreign money demand because there are costs of adjusting the initial stock of foreign money balances, e.g. equation (22b). When \( V_{12} > 0 \), the long run effect on the inflation tax is negative since foreign money balances fall but the initial gain offsets the long run loss. When \( V_{12} < 0 \), the long run effect is also positive. This is an important result because of the source-based characteristic of the inflation tax discussed above. The revenue from the foreign inflation tax accrues directly to the foreign government, which may give scope for additional inflation tax revenues without Laffer-style constraints abroad. Obviously, these would have to be weighted with the costs of inflation for the foreign resident in the foreign country.
4. CONCLUSIONS

This theoretical model provides insights about the short versus long run trade-offs and transitional dynamics of domestic and foreign monetary shocks by introducing the concept that, to switch into foreign currency, the individual incurs an adjustment cost. This device, besides being intuitively appealing, overcomes the problem of the degenerate transitional dynamics pointed out by Engel (1989). In addition, the model is a plausible positive framework for the analysis of the issue of the imperfect complement/imperfect substitute versus competitive characteristic of domestic and foreign currencies in terms of the liquidity services provided by entering directly in the utility function.

The main results of the paper can be summarized as follows. The way assets behave along the transitional path depends critically upon (i) the (imperfect) substitute/complement versus competitive aspect of domestic and foreign currency in the utility function, or the sign of $V_{12}$; and (ii) the relative sizes of the foreign inflation rate and the speed of adjustment towards the steady state. An increase in the domestic monetary growth rate induces an instantaneous decrease in domestic money balances, but the long run effect ultimately depends on the sign of $V_{12}$. Hence, the overall effect on the present discounted value of inflation tax revenue is also shown to depend on the sign of $V_{12}$. In particular, I show that the possible long run gains in domestic money holdings trade-off with the initial inflation tax effect, which may ultimately reduce the present discounted value of future inflation tax revenues. An analogous result for foreign inflation disturbances is presented, except that in this case the present discounted value of future foreign inflation tax revenues increases regardless of the sign of $V_{12}$. In deriving these effects, I fully take into account the general source-based property of the inflation tax, hence accounting for the implied effects of domestic holdings of foreign currency on the current account balance.

It may be of interest to introduce production and capital accumulation in the model in future work. The explicit consideration of uncertainty, and the possibility of direct estimation of the equilibrium dynamic paths in Figures 1 and 2 with real world data, also seem fruitful avenues for further research.

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APPENDIX

Linearizing equations (10, a, b, c) around the steady state equilibrium yields a $3 \times 3$ matrix of the form:

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\[
\begin{vmatrix}
  a_{11} & a_{12} & 0 \\
  0 & 0 & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{vmatrix}
\]

(A1)

where \( a_{11} = -m_c V_{11}/\lambda_c > 0; \ a_{12} = -m_c V_{12}/\lambda_c; \ a_{23} = \Phi_1^* > 0; \ a_{31} = -V_{12}; \ a_{32} = -V_{22} > 0; \ a_{33} = \delta + \pi^* > 0 \)

The determinant is \( a_{33}(a_{12}a_{31} - a_{32}a_{11}) \) which is negative as long as \( V(., .) \) is concave and the cost function \( \phi(.) \) is convex. The trace is \( (a_{11} + a_{33}) \) which is unambiguously positive. Thus, two roots are positive and one is negative.

Linearizing equation (8) around the steady state yields the expression

\[
b^* = \delta (b^* - b_x^*) - \phi'\Phi_x^*(\mu - \mu_c) - (\phi'\Phi_y^* - \phi'\pi^* + \pi^*)(m^* - m_c^*)
\]

(A2)

which, evaluated at the steady state \( \phi'(0) = 1 \), gives

\[
b^* = \delta (b^* - b_x^*) - \Phi_x^*(\mu - \mu_c) - \Phi_y^*(m^* - m_c^*)
\]

(A3)

Substituting equations (12b, c) into (A3), integrating the resulting expression and applying the condition in equation (11) gives equation (12d) in the text and (A12) below.

The steady state obtains when

\[
\dot{m} = m^* = \dot{b}^* = \dot{\mu} = 0
\]

and is characterized by the equations

\[
U'(c_c) = \lambda_c
\]

(A4)

\[
[V_1(m_c, m_c^*)/\lambda_c] = \delta + \sigma
\]

(A5)

\[
[V_2(m_{c}, m_c^*)/\lambda_c] = \delta + \pi^*
\]

(A6)

\[
\sigma = \pi_c
\]

(A7)

\[
\mu_c = \lambda_c
\]

(A8)

\[
\Phi_c^* = \pi^* m_c^*
\]

(A9)

\[
T = \sigma m_c
\]

(A10)

\[
y_c - c_c + \delta b_c^* - \pi^* m_c^* = 0
\]

(A11)

\[
b_c^* - b_c^* = [\Omega/(\gamma - \delta)] [m_0 - m_c]
\]

(A12)

These are nine equations in the unknowns \( c_c, m_c, m_c^*, \Phi_c^*, \pi_c, b_c^*, \lambda_c, T \) and \( \mu_c \); given \( m_0, b_0^*, \sigma \) and \( \pi^* \).

The comparative dynamics results are all obtained from the solution of (A4)–(A12). Specifically, the solution for an increase in the domestic monetary growth rate or the foreign inflation rate is obtained by substituting (12b) and (A12) using (15) into (A4)–(A7) yielding the effects on \( m_c, m_c^* \) and
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$c_c$ given by:
\[ \frac{dc_c}{d\sigma} = - \left\{ U'V_{22} - U''a_{33}U'\left[ \pi^* - (\delta \Omega/\Psi^* (\gamma - \delta)) \right]\right\}/D \]  
(A13)
\[ \frac{dm_c}{d\sigma} = - U'V_{12}/D \]  
(A14)
\[ \frac{dc_c}{d\pi^*} = - U''V_{12}\left[ \pi^* - (\delta \Omega/\Psi^* (\gamma - \delta)) \right]/D \]  
(A15)
\[ \frac{dm_c}{d\pi^*} = \left\{ U'V_{12} + U''V_{22}m_c^* (\delta + \sigma) - U'V_{12}m_c^*a_{33} + U''(\delta + \sigma)U'\left[ \pi^* - (\delta \Omega/\Psi^* (\gamma - \delta)) \right]\right\}/D \]  
(A16)
\[ \frac{dm_c}{d\pi^*} = \left\{ U''V_{11}m_c^*a_{33} - U''V_{12}m_c^* (\delta + \sigma) - U'V_{11}\right\}/D \]  
(A17)
\[ \frac{dc_c}{d\pi^*} = \left\{ V_{11}V_{22}m_c^* + U'\left[ \pi^* - (\delta \Omega/\Psi^* (\gamma - \delta)) \right] - (V_{12})^2m_c^*\right\}/D \]  
(A18)
\[ \frac{dm}{d\pi} = \frac{dm_c}{d\pi} - (1/\Psi^*) \frac{dm_c}{d\pi}, \text{ for } i = \sigma, \pi^* \]  
(A19)

where
\[ D \equiv U'\left[ \pi^* - (\delta \Omega/\Psi^* (\gamma - \delta)) \right] \left[ V_{12}(\delta + \sigma) - V_{11}a_{33} \right] \]  
(A20)

NOTES

1 Giovannini and Turtelboom (1994) provide a useful review of the related currency substitution literature. I assume that the adjustment cost only applies to foreign currency and not to the domestically traded bond (denominated in foreign currency). The case I focus on is a potential situation where acquiring foreign currency is more costly than acquiring domestically traded bonds. For example, a domestic bank may offer a bond denominated in foreign currency at no additional cost to achieve a domestic saving objective, but for an individual to acquire foreign currency, he or she either incurs an additional transaction fee or it is legally restricted by the central bank. In addition, the foreign currency does not bear interest whereas the bond does, thus adding to the differential cost. The reader interested in a model with quadratic costs to holding domestically traded (foreign) bonds, without foreign currency, may read Turnovsky (1985).

2 In a framework designed to examine the effects of currency substitution on the real exchange rate, Engel (1989), who assumed that the domestic and foreign currencies are competitive in utility, pointed to the anomaly generated by Calvo’s (1985) assumption that the two currencies are (imperfect) substitutes/complements in utility; see also Rogers (1990) for further references.

3 The small open economy assumption is not restrictive in studying this problem and has the advantage of giving exchange rate determinacy. The two-country model may present indeterminacy of the exchange rate when the currencies are perfect substitutes in utility, see for example Weil (1991).

4 I have suppressed time subscripts where time dependence is more obvious, and assumed the convention that a dot over a variable indicates its derivative with respect to time, a subscript c on a variable indicates its steady-state constant value, (exp) indicates the exponential operator, a starred variable indicates a foreign variable, and lower case variables are real variables unless otherwise noted.

5 The cost function I assume is similar to the well-known cost of adjustment function for investment in models with productive capital, e.g. Hayashi (1982). There is a technical reason for assuming that the cost function is convex, that is, it guarantees that the economy presents saddlepoint dynamics and thus the dynamic adjustment
The result that, in a bond economy, the transitional dynamics degenerate has been pointed out by Engel (1989); see also Weil (1991). Finally, a decrease in the parameter \( h \) could be interpreted as reflecting financial innovation and adaptation, e.g. Guidotti and Rodriguez (1992) and Guidotti (1993); however, the qualitative results in this paper are not very sensitive to the magnitude of this parameter as long as \( h \neq 0 \).

For any value of \( \epsilon \in (0, 1) \), as long as \( \beta = 1 \), i.e. the elasticity of substitution between the currencies is infinity, the currencies are perfect substitutes in utility. This does not impose the condition that \( V_{12} \rightarrow 0 \) for perfect substitutability, in many cases the currencies are perfect substitutes and \( V_{12} < 0 \), say \( \beta = 1 \) and \( \alpha = 1/2 \), or \( V(, ,) = \ln(m + m^*). \) Canzoneri and Diba (1993) work with a more general utility function where consumption and money are non-separable. My assumption of separability of consumption and money is made for analytical purposes basically to narrow the focus on the attributes of currencies in utility.

For analytical simplicity I assume that there is no domestic bond. In a small open economy a domestic bond would pay the exogenously given interest rate (as the traded bond), but would be denominated in domestic currency, thus subject to an instantaneous wealth effect as noted below. I also assume that foreign residents do not hold domestic currency and point out below how relaxing this assumption impacts on the analysis.

The current account equation (8) includes the change in the real stock of foreign currency by the definition of \( \delta (\Phi - \pi^* m^*) \) in (1)-(1a). If foreigners held domestic currency, the right-hand side of (8) would include a term denoting the revenue received from the inflation tax from abroad.

See Turnovsky and Bianconi (1992) and Bianconi (1995) for a distinction between source and residence-based taxation in this class of model.

The level and the rate of change of the nominal exchange rate in this model are determinate since the foreign and domestic price levels and rates of change are determined. See for example Weil (1989, 1991) for models of this class where the exchange rate is indeterminate.

The two polar cases that remain are the following. If the elasticity of substitution between the currencies is infinity, or \( \beta = 1 \), the slope of the locus is zero and the manifold is infinitely elastic (horizontal) at \( m^* \). On the other extreme, as \( \beta \rightarrow \infty \) and \( V_{12} \rightarrow \infty \) the elasticity of substitution approaches zero and the locus is perfectly inelastic (vertical) at \( m^* \).

In cases (i) through (iv), if the foreign resident held domestic currency, the slope of the locus would become smaller or larger in absolute value, however this would not change the main qualitative implications of the dynamic adjustment.

In order to sign these effects unambiguously I have parameterized the equilibrium with \( \alpha = 0.5 \), \( \beta = 0.25 \) for \( V_{12} > 0 \) and \( \beta = 1.25 \) for \( V_{12} < 0 \); \( h = 5 \), and evaluated the effects around \( \sigma = \pi^* = 0.04 \), \( \delta = 0.02 \) and \( y_0 = 1 \). Under these and a range of parameter values, the speed of adjustment is always small in magnitude, implying that \( \gamma > -\pi^* \) is the relevant case, see for example Barro and Sala-i-Martin (1992) for similar results regarding the low magnitude of the speed of adjustment. The results are also qualitatively the same under a range of plausible parameter values.

Note here that if there were an adjustment cost to switching into traded bonds, then \( db^* \) would still be zero, but its marginal adjustment cost would be another jump variable increasing the dimension of the dynamical system. On the other hand, if there were a domestic bond (which would also increase the dimension of the dynamical system), the initial wealth effect would apply to the sum of domestic money and bonds on the portfolio of the domestic resident.

Imrohoroglu (1996) relates the result of an upward sloping monetary Laffer curve to the case where the elasticity of currency substitution is small. His elasticity of
currency substitution in my parameterization is $\beta/(\beta - 1)$, which corresponds to the case where $\beta < 1$, i.e. $V_1 > 0$. In his specification the relative weights of the currencies in utility differ, but in my simulations the results were insensitive to this alternative. See also Imrohoroglu (1994) for empirical evidence on the magnitude of the elasticity of currency substitution for US–Canada.

REFERENCES


