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On dynamic real trade models

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Abstract

This paper examines the steady-state implications of differences in tax policies in an overlapping generations, two-country, two-good, dynamic model of international trade. It is also shown that, in the overlapping generations framework, production diversity along with factor-price equalization obtains in the long run.

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1. Introduction

The purpose of this paper is to show some basic new results in the class of dynamic real trade models. Earlier some authors analyzed trade theory from a dynamic perspective, e.g. Oniki and Uzawa (1965) and Stiglitz (1970); more recently, Baxter (1992) has built on that tradition by analyzing government expenditure policy in this framework. Here I specify the saving–investment demand-side decision according to the life-cycle paradigm. There are two main new results established in the paper: (i) it is shown that differences in tax rates may be one of the determinants of the long-run pattern of trade among countries; and (ii) in the life-cycle framework, long-run factor-price equalization holds along with diversity of production; contrary to the case of specialization in the infinitely lived representative agent models of Baxter (1992) and Stiglitz (1970).[1] The second result emerges from the fact that Stiglitz (1970) and Baxter (1992) model the demand side according to the infinitely lived representative agent paradigm. I use the life-cycle saving model which introduces heterogeneity among agents. It is established here that in the life-cycle saving model, heterogeneity implies that long-run production diversity obtains. In contrast, in the infinitely lived

[1] More recently, Matsuyama (1988a, b) has studied dynamic trade models with an explicit life-cycle saving–investment decision but he restricts the analysis to the case of a small open economy. See also Eaton (1987). In particular, this paper draws on the recent contributions by Galor and Lin (1989) and Galor (1992).

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representative agent models of Baxter (1992) and Stiglitz (1970) long-run specialization obtains.[2]

2. The model

Consider a competitive world consisting of two countries, D and F, each producing a pure consumption good, X_t, and a pure capital good, Y_t, traded over every period t. The two goods are produced using labor, $L_t^{i,j}$ and capital, $K_t^{i,j}$, for $i = X, Y$; and $j = \mathrm{D}, \mathrm{F}$. The labor endowment in each country is fixed. The countries are engaged in free trade in goods. There is neither international labor mobility nor international borrowing and lending. Without loss of generality, capital is assumed to depreciate fully each period. Perfect foresight is assumed throughout. Technology in each country,[3] in each sector, consists of time-invariant constant returns to scale production functions which imply $X_t^j = L_t^{x,j} f_x(k_t^{x,j})$ and $Y_t^j = L_t^{y,j} f_y(k_t^{y,j})$, where $k_t^{i,j} = (K_t^{i,j}/L_t^{i,j})$ is the capital–labor ratio in sector i and country j, and $f_i(\cdot)$ is the production function in sector i, common to both countries, which is twice continuously differentiable, positive, increasing, and strictly concave, or $f_i(k^{i,j}) > 0$; $f_i'(k^{i,j}) > 0$; $f_i''(k^{i,j}) < 0$. Also $f(\cdot)$ satisfies the usual Inada conditions. In each country, the relative factor intensities may be summarized as: (i) if $k^{x,j} > k^{y,j}$, then the consumption good is more capital intensive than the capital good; (ii) if $k^{x,j} < k^{y,j}$, then the capital good is more capital intensive than the consumption good. Factor intensity reversals are ruled out throughout the paper, by assumption. In each country, both goods are produced with K and L perfectly mobile across sectors. If both goods are produced, the zero profit conditions for firms yield

$$r_t^j = p_t^k f_x'(k_t^{x,j}) = f_y'(k_t^{y,j}) , \tag{1a}$$

$$\begin{aligned} w_t^j &= p_t^j[f_x(k_t^{x,j}) - f_x'(k_t^{x,j})k_t^{x,j}] \\ &= [f_y(k_t^{y,j}) - f_y'(k_t^{y,j})k_t^{y,j}] , \end{aligned} \tag{1b}$$

where r is the rental rate, w is the wage rate, and p is the price of the consumption good in terms of the price of the capital good, the latter normalized to unity, i.e. the capital good is assumed to be the numeraire. It is straightforward to show that $k_t^{i,j} = k^{i,j}(\Omega_t^j)$, where Ω_t^j is the wage–rental ratio and $k(\cdot)$ is strictly decreasing. From (1a), if both goods are produced (I assume this to be the case throughout the paper), one then obtains $p_t^j(\Omega_t^j) = f_y'[k^{y,j}(\Omega_t^j)]/f_x'[k^{x,j}(\Omega_t^j)]$, which yields $\Omega_t^j = \Omega(p_t^j)$, with $\Omega(\cdot)$ strictly increasing. In turn, the rental and wage rates are uniquely determined as a function of the relative price $w_t^j = w(p_t^j)$ and $r_t^j = r(p_t^j)$. Finally, the per worker production of each good, in each country, is given by

$$\begin{aligned} X_t^j/L^j = x_t^j &= [(k_t^j - k_t^{y,j})/(k_t^{x,j} - k_t^{y,j})] \\ &\times f_x[k^{x,j}(\Omega(p_t^j))] = x(p_t^j, k_t^j) , \end{aligned} \tag{2a}$$

[2] The paper is also intended to provide an extension of Diamond (1965) for the case of non-joint production in two distinct sectors: a consumption good sector and a capital good sector, see, for example, Sibert (1990).

[3] The production side of the model is based on the two-sector model of growth as in Uzawa (1961, 1963).

$$Y_t^j/L^j = y_t^j = [(k_t^{x,j} - k_t^j)/(k_t^{x,j} - k_t^{y,j})] \times f_y[k^{y,j}(\Omega(p_t^j))] = y(p_t^j, k_t^j)\,, \tag{2b}$$

where L^j is aggregate labor in country j, and $k_t^j = K_t^j/L^j$ is the aggregate capital–labor ratio in country j.

The demand side consists of life-cycle consumption–saving behavior. Every period t, L^j individuals are born in country j. Identical individuals live for two periods. In the first period, the young generation works at the competitive wage w_t^j, and allocates its human wealth between consumption and saving. In the second period, the now old generation retires consuming all after-tax savings plus a transfer from the government. The problem is to choose c_{1t}^j and s_t^j to

$$\max u(c_{1t}^j) + \beta^j u(c_{2t+1}^j) \tag{3}$$

subject to $s_t^j = w_t^j - p_t^j c_{1t}^j$ and $p_{t+1}^j c_{2t+1}^j = r_{t+1}^j (1 - \tau^j) s_t^j + T_{t+1}^j$, where $0 < \beta < 1$ is the discount factor, c_1 is per labor consumption when young, c_2 is per labor consumption when old, s is saving, $0 < \tau < 1$ is the tax rate on interest income, and T is the per labor lump-sum transfer.[4] The function $u(\cdot)$ is a twice continuously differentiable, monotonically increasing, and quasi-concave utility function, or $u'(\cdot) > 0$, $u''(\cdot) < 0$, satisfying the usual Inada conditions. Solution of (3) implies a smooth savings function of the form

$$s_t^j = s(w_t^j, r_{t+1}^j, p_{t+1}^j, p_t^j, T_{t+1}^j; \tau^j, \beta^j)\,, \tag{4}$$

where $\partial s/\partial w_t^j > 0$ under the assumption that consumption in both periods is a normal good, $\partial s/\partial \tau^j < 0$ and $\partial s/\partial \beta^j < 0$. I shall assume throughout the paper that $\partial s/\partial r_{t+1}^j \geq 0$. If $\partial s/\partial r_{t+1}^j > 0$, this implies that the substitution effect dominates, or alternatively that the elasticity of substitution between consumption in the two periods is greater than one. If $\partial s/\partial r_{t+1}^j = 0$, this implies that the elasticity of substitution is equal to one, or alternatively the logarithmic utility function. The government, in each country, is assumed to transfer all receipts back to individuals according to the rule $T_t^j = r_t^j \tau^j s_{t-1}^j$.

3. Dynamic trade equilibrium, steady state, and pattern of trade

The factor returns and the transfer may be substituted into (4) yielding the equilibrium saving function. If $\partial s/\partial r_{t+1}^j > 0$, then $s_t^j = s(\cdot) = S(p_t^j, p_{t+1}^j; \tau^j, \beta^j)$, where $\partial S/\partial p_t^j = [(\partial s/\partial w)(\partial w/\partial p_t^j) + (\partial s/\partial p_t^j)]$, $\partial S/\partial p_{t+1}^j = \{-p_t[(\partial c_1/\partial p_{t+1}^j) + (\partial c_1/\partial r)\ \ (\partial r/\partial p_{t+1}^j)]\}$, $\partial S/\partial \tau^j < 0$ and $\partial S/\partial \beta^j < 0$. If $k^x > k^y$ and $\partial s/\partial r_{t+1}^j > 0$, then $\partial S/\partial p_{t+1}^j > 0$ and the sign of $\partial S/\partial p_t^j$ is

[4] Diamond (1970) considers a similar tax scheme in a one-sector overlapping generations model of a closed economy. Another possibility would be to transfer in lump-sum fashion to the young generation; see, for example, Atkinson and Stiglitz (1980, Lecture 8). I chose the Diamond (1970) scheme because it conveniently captures the distortionary effect of taxes.

ambiguous. If $k^x < k^y$ and $\partial s/\partial r^j_{t+1} > 0$, then $\partial S/\partial p^j_{t+1} < 0$ and $\partial S/\partial p^j_t > 0$. In equilibrium, investment equals saving in each country, or $k^j_{t+1} = S(p^j_t, p^j_{t+1}; \tau^j, \beta^j)$. The goods market equilibrium in the jth country's capital good sector requires, by (2b), that $S(p^j_t, p^j_{t+1}; \tau^j, \beta^j) = y(p^j_t, k^j_t)$. The world dynamic equilibrium is then obtained by considering each country's saving–investment equilibrium (since international borrowing and lending does not exist) and the equilibrium condition in the world capital good sector, or

$$k^{\mathrm{D}}_{t+1} = S(p_t, p_{t+1}; \tau^{\mathrm{D}}, \beta^{\mathrm{D}}) , \tag{5a}$$

$$k^{\mathrm{F}}_{t+1} = S(p_t, p_{t+1}; \tau^{\mathrm{F}}, \beta^{\mathrm{F}}) , \tag{5b}$$

$$S(p_t, p_{t+1}; \tau^{\mathrm{D}}, \beta^{\mathrm{D}}) + S(p_t, p_{t+1}; \tau^{\mathrm{F}}, \beta^{\mathrm{F}}) = y(p_t, k^{\mathrm{D}}_t) + y(p_t, k^{\mathrm{F}}_t) , \tag{5c}$$

with k^{D}_0 and k^{F}_0 exogenously given.[5]

The steady-state trade equilibrium consists of the fixed point $\{k^{\mathrm{D}}, k^{\mathrm{F}}, p\}$ which satisfy Eqs. (5a)–(5c). It is clear that, under the implicit assumption that both goods are produced, the economies above satisfy the Stolper–Samuelson and the Rybczynski theorems. Also, if the two countries are equal in every respect, the trade equilibrium coincides with the autarky equilibrium [by inspection of (5)]. The steady-state trade equilibrium satisfies the following propositions:[6]

Proposition 1. In the two-country steady-state equilibrium, assume: (i) $k^{x,j} > k^{y,j}$, *or the consumption good is capital intensive*; (ii) $\tau^{\mathrm{D}} > \tau^{\mathrm{F}}$, *or the tax rate in the domestic country is greater than in the foreign country*; (iii) *countries are identical in all other respects. Then, the low-tax (foreign) country exports the capital-intensive good while the high-tax (domestic) country exports the labor-intensive good.*

Proposition 2. In the autarky steady-state equilibrium, assume (i), (ii) *and* (iii), *as in Proposition 1. Then, if trade opens up, in the two-country steady-state equilibrium*: (i) *the wage rate decreases and the rental rate increases in the low-tax (foreign) country*; (ii) *the wage rate increases and the rental rate decreases in the high-tax (domestic) country.*

Proposition 3. In the two-country steady-state equilibrium, assume (i) *and* (iii) *as in Proposition 1, and* (ii). $\tau^{\mathrm{D}} \neq \tau^{\mathrm{F}}$, *or the tax rate in the domestic country is different from that in the foreign country*; (iv) *both goods are produced in each country. Trade equalizes factor prices in both countries.*

The result of Proposition 3 illustrates one of the basic features of the two-sector life-cycle framework. In the infinite-horizon single-agent model of Stiglitz (1970) and Baxter (1992),

[5] The equilibrium in every period requires that: (i) production of the capital good equals aggregate saving; (ii) production of the consumption good equals aggregate consumption; (iii) supply and demand for capital and labor are equal in both sectors.

[6] Proofs of Propositions 1, 2, and 3 are available from the author upon request.

say, trade leads to long-run specialization of at least one of the countries inhibiting long-run factor-price equalization. In the two-sector life-cycle economy, diversification in production in both countries is feasible and long-run factor price equalization holds.[7] Propositions 1–3 establish the result that differences in tax rates on interest income are one of the determinants of the pattern of trade in the long run. The result is conceptually consistent with the traditional Hecksher–Ohlin theory of trade, even though that theory emphasizes differences in factor endowments. In this sense, tax policy defined according to Parente and Prescott (1994) may be one of the determinants of the pattern of trade in the long run.

For the analysis of the production possibilities frontier (PPF), assume, without loss of generality, a one-country world under autarky. The steady-state PPF may be conveniently summarized by the expression (see, for example, Jones, 1965), $L = L^x + L^y = (L^x/X)X + (L^y/Y)Y$. Its slope follows directly as $\mathrm{d}Y/\mathrm{d}X|_{L^x+L^y=L} = -f_y(k^y)/f_x(k^x)$. It is easy to show that the steady-state PPF is nonlinear. From the first-order conditions of problem (3), we have, in the steady state, $(1-\tau)f_y'(k^y) = [u'(c_1)/\beta u'(c_2)]$. This implies that the solution for the capital–labor ratio in the capital good sector, k^y, is not independent of consumer demands, or alternatively k^y is not independent of the composition of output between X and Y. In effect, as the composition between X and Y varies, the capital–labor ratios also vary and the steady-state PPF is nonlinear. In a recent paper, Baxter (1992) shows that in a two-sector model of growth with an infinitely lived representative agent, the steady-state PPF is linear, implying that at least one country must specialize in the production of at least one good. I have just shown above that this is not the case in the overlapping generations model. How can one account for this difference? The technical answer is that in the infinitely lived representative agent model the solution for the steady-state capital–labor ratio in the capital good sector, k^y, is independent of consumer demands, and given by $(1-\tau)f_y'(k^y) = (1/\beta)$. There is a version of the dynamic nonsubstitution theorem of Mirrlees (1969) at work here. Alternatively, k^y is independent of the composition of output between X and Y and the steady-state PPF is linear.

Intuitively, what accounts for the difference is that in the overlapping generations model there are two distinct consumers that are able to trade with each other at any point in time. In effect, they are able to borrow from and lend to each other and this is reflected in the dependence of k^y on $[u'(c_1)/u'(c_2)]$, which is the marginal rate of substitution between consumption of the young and old in the steady state. The overlapping generations economy does not satisfy the dynamic nonsubstitution theorem. In the case of the infinitely lived representative agent, he/she is not able to borrow and lend at any point in time because, in equilibrium, net private debt must be zero. In other words, there is only one consumer, which implies that the marginal rate of substitution must be one, i.e. the consumer perfectly smooths its consumption stream. Even though the infinitely lived representative consumer framework is a very useful setting for the analysis of a variety of intertemporal dynamic problems, it is not as suitable for the analysis of, say, consumer issued debt. In this specific problem the absence of consumer heterogeneity implies that the economy must specialize in the production of at

[7] Note that propositions 1, 2, and 3 also hold for the case when $\tau^D = \tau^F$, but $\beta^D \neq \beta^F$; see, for example, Galor and Lin (1989).

least one good in the steady state. Introducing heterogeneous consumers allows for the steady state diversification of production.

One important issue that arises in the class of overlapping generations models regards efficiency; see, for example, Galor and Ryder (1991). In the economies presented in (1)-(10), the solution for k^y is given by $f_y'(k^y) = r[p(\tau, \beta)] = [u'(c_1)/\beta u'(c_2)(1-\tau)]$. If $k^y > k^{y*}$, or alternatively $f_y'(k^y) = [u'(c_1)/\beta u'(c_2)(1-\tau)] < 1$, then the economy is overinvesting relative to the golden rule and the steady-state equilibrium is dynamically inefficient. If $k^y < k^{y*}$, or alternatively $f_y'(k^y) = [u'(c_1)/\beta u'(c_2)(1-\tau)] > 1$, then the economy is underinvesting relative to the golden rule and the steady-state equilibrium is dynamically efficient.

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