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Monetary growth innovations in a simple cash-in-advance asset-pricing model

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This paper contributes to the existing literature in the rational expectations cash-in-advance asset-pricing general equilibrium model in two directions: first, we show that by allowing a variable velocity of circulation, an increase in the conditional variance of the money growth process triggers an increase in the demand for money relative to stocks and bonds with a consequent reduction in stock and bond prices, which is the opposite of the current result in the literature with constant unitary velocity; second, we explicitly show how the sequential markets and specific timing of information fits exactly into the notion of irreversibility in the sense that when faced with uncertainty, agents might find optimal to delay decisions in order to wait for the arrival of new information.

1. Introduction

We present a simple general equilibrium stochastic model where money enters via a cash-in-advance constraint on consumption goods. The basic framework, which derives from the many contributions of Lucas (1978, 1980, 1982), has been extended by Lucas (1984) and Svensson (1985a) to allow for a demand for money consistent with non-unitary velocity of circulation. More recently, Giovannini (1989) extended the Lucas and Svensson framework by allowing time-varying distributions of the state and examining the effects of expected future disturbances in the state. However, Giovannini (1989) restricted his analysis to states where monetary velocity is unity with probability one. Our paper examines the role of increased future monetary volatility, with time-varying distributions, relaxing the unitary velocity assumption and therefore allowing for a more general demand for money which combines the transactions motive, implicit in the cash-in-advance constraint, the precautionary motive, implicit in the liquidity services provided by money [as emphasized by Giovannini (1989)], and a store-of-

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value motive that arises when money is expected to yield a positive dividend. Specifically, we extend the stochastic structure used by Giovannini (1989) allowing the cash-in-advance constraints to bind or not bind probabilistically. We are able to examine the interactions between the degree of risk and the probabilistic outcome with respect to the cash-in-advance constraint. We take this opportunity to analyze nominal (monetary) disturbances.[1]

We offer an interpretation of the cash-in-advance framework based on the irreversibility that arises in the asset holdings decision consistent with the timing of events introduced by Svensson (1985a). The agent is in effect committed to his decision, therefore losing an option value with respect to the relative degrees of liquidity among the assets. In turn, the arrival of new information plays a decisive role in the results obtained, in the sense that it may be worth not to commit and to maintain an open option to some future commitment when new information is available.

The specific contribution of this paper to the existing literature is twofold. First, by extending Giovannini's (1989) analysis to allow for a variable velocity of circulation, we show that his results with respect to a change in the volatility of monetary growth only apply to the case when the cash-in-advance constraint binds every period. In other words, we show that with variable velocity, an increase in the conditional variance of the money growth process triggers an increase in the demand for money relative to stocks and bonds with a consequent reduction in stock and bond prices, which is the opposite result originally obtained by Giovannini (1989).[2] Second, we explicitly show how to interpret the cash-in-advance framework, with the sequence of markets proposed by Svensson (1985a), as introducing an element of irreversibility into the representative agent's decision to optimally allocate consumption, money, stocks, and bonds. In particular, we show that since money provides full liquidity and stocks and bonds do not, a store-of-value demand for money arises when the agent receives a signal that the cash-in-advance constraint will not bind. However, because the agent is uncertain about the future cash-in-advance constraint being binding or not,

[1]Authors such as LeRoy (1984), Stulz (1986), and Danthine et al. (1987) have used a related general equilibrium framework but adopted the route of entering money directly in the utility function.

[2]Recently, Hodrick et al. (1989) have concluded, on the basis of a calibration procedure, that there is little practical gain in using cash-in-advance models that allow for a variable velocity of circulation. However, Giovannini and Labadie (1989) show that their result is sensible to the sample period. Our analysis show that, in this class of models, restricting the analysis to a constant unitary velocity of circulation may lead to wrong predictions of the effect of a change in the volatility of the exogenous driving process on stock and bond prices. The results we obtain are in the spirit of the empirical work of Finn et al. (1990) where it is found that the model presented in this paper is the only one, in its class, in which monetary effects improve the explanation of asset returns. Also, Danthine et al. (1987), who adopted the route of entering money directly in the utility function, found the level and variability of velocity to be sensible to changes in the exogenous driving process.

it might be optimal to wait for the arrival of new information while leaving relative asset demands unchanged.[3]

The paper is organized as follows. In section 2 we present the basic model and in section 3 we solve the model deriving the equilibrium. Section 4 presents the stochastic structure that allows the cash-in-advance constraint to be binding or not probabilistically. Section 5 presents the basic analytical results while section 6 gives intuition about the interpretation of the framework from the point of view of irreversibility. Finally, section 7 presents summary and conclusions.

2. Macroeconomic structure

Consider a discrete time stochastic model of a closed monetary economy with one non-storable good, without capital, inhabited by households, firms, and government. Firms are assumed to produce an exogenous constant level of real output. The model follows closely the set up of Svensson (1985a) and Giovannini (1989):

(i) *Households.* Every period, a representative household solves a choice-theoretic problem in order to optimally allocate his/her total wealth between consumption, stock holdings, and money holdings. The problem is

$$\max_{\{c_t, M_{t+1}, z_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1a}$$

subject to

$$(M_{t+1}/P_t)+(Q_t/P_t)z_{t+1}$$
$$\leqq (M_t/P_t - c_t)+(Q_t/P_t + y_t)z_t + (\omega_t - 1)(\tau_t/P_t), \tag{1b}$$

$$c_t \leqq M_t/P_t, \tag{1c}$$

$$z_0 = 1, \quad M_0 > 0 \text{ given}, \tag{1d}$$

where

[3]One of the original contributions that led to the analysis of the irreversibility effect (mainly in investment and natural resources) is due to Marschak (1949) whose main concern was the commitment to assets which would then be 'frozen', as opposed to 'liquid' in the case of no commitment. Other authors have associated the liquidity of an asset, as opposed to the perfectly liquid asset (money), with flexibility which also fits in our interpretation of the cash-in-advance framework with the sequence of markets proposed by Svensson (1985a). A recent extensive analysis of the issue of flexibility and its relation to liquidity and irreversibility is found in Jones and Ostroy (1984).

$0<\beta<1$ is a constant discount factor,
E_0 = expectation operator conditional on information at $t=0$ with respect to the probability distribution of $\{c_t\}_{t=0}^{\infty}$ to be defined below,
c_t = real consumption at time t,
P_t = price level at time t,
Q_t = money price of stock (share) at time t,
M_t = nominal money holding at time t,
z_t = quantity of perfectly divisible stock (share) held at time t,
y_t $\equiv y$ = constant real output equal to dividend rate,
ω_t = gross rate of monetary growth at time t,
$(\omega_t-1)\,\tau_t$ = lump sum monetary transfer (tax) at time t,
$U(\cdot)$ = utility function with $U'(\cdot)>0$, $U''(\cdot)<0$, $U'(0)=\infty$, $U'(\infty)=0$.

Eq. (1b) is a flow budget constraint, in real terms, where the right-hand side shows money leftovers from consumption plus value and dividends on stock holdings plus the monetary transfer being allocated between future money and stock holdings (left-hand side). Eq. (1c) is the cash-in-advance constraint on consumption goods and (1d) is the initial wealth. Households face an information constraint and a decentralized sequence of markets, as in Svensson (1985a), shown in fig. 1. It implies that once a decision at period t to carry a certain amount of cash balances to $t+1$ is made, it is irreversible in the sense that, when facing the goods market in period $t+1$, the consumer will be committed to consume only up to that amount,

(ii) *Government.* The role of the government is to transfer(tax) at the stochastic gross rate of growth of money ω_t according to the rule

$$\tau_{t+1}=\omega_t\tau_t. \tag{2}$$

The state of the macroeconomy in period t is defined as $s_t\equiv\omega_t$, where the process ω_t is Markov with $\Pr\{\omega_{t+1}\leqq\omega'|\omega_t=\omega\}=H(\omega',\omega)$ and $H(\cdot,\cdot)$ has conditional density (assumed to exist) $h(\omega',\omega)$.

3. Macroeconomic equilibrium

The equilibrium is characterized by forming the Lagrangean[4]

$$L\equiv\mathrm{E}\left[\sum_{t=0}^{\infty}\beta^t\{U(c_t)+\lambda_t[(M_t/P_t-c_t)+(Q_t/P_t+y)z_t\right.$$

[4]Existence results for models related to the one used in this paper may be found in Townsend (1987) and Lucas and Stokey (1987).

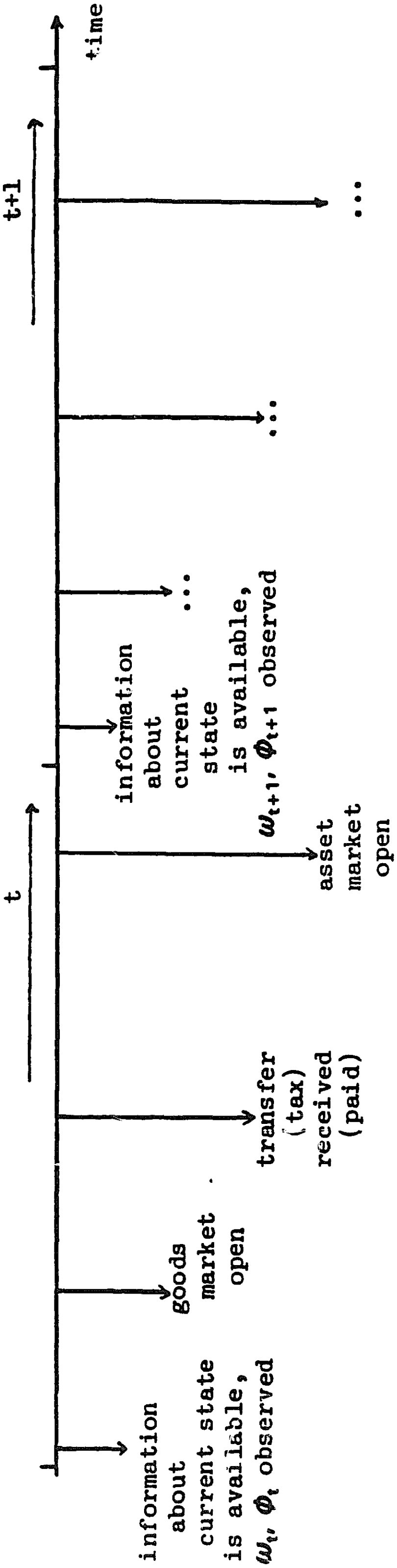

Fig. 1. Information availability and sequence of markets.

$$+(\omega_t-1)\tau_t/P_t-M_{t+1}/P_t-(Q_t/P_t)z_{t+1}]+\mu_t(M_t/P_t-c_t)\}\,|\cdot\Big]. \tag{3}$$

The equilibrium is then a rational expectations equilibrium defined by

$$L_c\to \quad U'(c_t)-\lambda_t-\mu_t=0, \tag{4a}$$

$$L_M\to \quad \lambda_t/P_t=\beta\mathrm{E}[(\lambda_{t+1}+\mu_{t+1})/P_{t+1}|\cdot], \tag{4b}$$

$$L_z\to \quad \lambda_t Q_t/P_t=\beta\mathrm{E}[\lambda_{t+1}([Q_{t+1}/P_{t+1}]+y)|\cdot], \tag{4c}$$

$$\mu_t\geqq 0, \quad M_t/P_t\geqq c_t, \tag{4d}$$

$$\text{goods market equilibrium,} \quad c_t=y, \tag{4e}$$

$$\text{money market equilibrium,} \quad M_{t+1}=\tau_{t+1}=\omega_t\tau_t=\omega_t M_t, \tag{4f}$$

$$\text{shares market equilibrium,} \quad z_t=1, \tag{4g}$$

$$\lim_{t\to\infty}\beta^{t-1}\,\mathrm{E}[(\mu_t+\lambda_t)M_t/P_t|\cdot]=0, \tag{4h}$$

$$\lim_{t\to\infty}\beta^{t-1}\,\mathrm{E}[(\mu_t+\lambda_t)(Q_t/P_t)z_t|\cdot]=0, \tag{4i}$$

where λ_t is the (positive) Lagrange multiplier on budget constraint (1b) and μ_t is the (non-negative) Lagrange multiplier on budget constraint (1c); λ_t is the marginal utility gain of a marginal increase in wealth while, μ_t is the marginal gain from a marginal relaxation of the cash-in-advance constraint (1c).

The definition of equilibrium is the standard definition of a stationary stochastic rational expectations equilibrium [see Brock (1982)] and it consists of a set of initial conditions $\{z_0=1, M_0>0\}$, money supply rule (2), the endogenous choice variables $\{c_t, M_{t+1}, z_{t+1}\}_{t=0}^{\infty}$, and prices of goods and assets $\{P_t, Q_t\}_{t=0}^{\infty}$ all of which satisfy: (i) given the pricing functions and the money supply rule, (4a)–(4d) solve the agent's maximization problem (1) for consumption, money holdings and asset holdings and the subjective probabilities are equal to the objective probabilities for all $t=1,2,\ldots$; (ii) the competitive markets for goods, money and shares clear, according to (4e)–(4g), for all $t=1,2,\ldots$; (iii) the transversality conditions at infinity (4h)–(4i) are satisfied for all $t=1,2,\ldots$. Eqs. (4a)–(4c) together with the market clearing conditions (4e)–(4g) solve for the equilibrium purchasing power of money, $1/P_t$, the marginal utility of wealth, λ_t, and the marginal liquidity service of money, μ_t given the respective quantities. The asset price may be solved separately by a recursion of (4c).[5]

The general explicit solution for the endogenous prices is obtained as in Svensson (1985a). If $h_{t-1}(M_0) \leqq (y/\beta \mathrm{E}[1/P_{t+1}|\cdot])$ for all possible realizations of $\mathrm{E}[\cdot|\cdot]$, then the liquidity constraint (1c) is binding almost surely (a.s.) such that

$$1/P_t = c_t/h_{t-1}(M_0) = y/h_{t+1}(M_0), \tag{5a}$$

$$\lambda_t = [\beta U'(y) h_{t-1}(M_0)/y] \mathrm{E}[1/P_{t+1}|\cdot], \tag{5b}$$

$$\mu_t = U'(y) - [\beta U'(y) h_{t-1}(M_0)/y] \mathrm{E}[1/P_{t+1}|\cdot] > 0, \tag{5c}$$

where $h_{t-1}(M_0) \equiv M_0 \prod_{j=0}^{\infty} \omega_{t-1-j}$ is the history of past monetary growth rates which determines the current money stock. If $h_{t-1}(M_0) > (y/\beta \mathrm{E}[1/P_{t+1}|\cdot])$ for all possible realizations of $\mathrm{E}[\cdot|\cdot]$, then the liquidity constraint (1c) is nonbinding a.s. such that

$$1/P_t = \beta \mathrm{E}[1/P_{t+1}|\cdot], \tag{5d}$$

$$\lambda_t = U'(y), \tag{5e}$$

$$\mu_t = 0. \tag{5f}$$

In both cases, the stock price is given by

$$Q_t/P_t = (\beta y/\lambda_t) \sum_{j=0}^{\infty} \beta^j \mathrm{E}[\lambda_{t+1+j}|\cdot] \quad \text{for} \quad \mu_t > 0 \text{ or } \mu_t = 0. \tag{5g}$$

One comment on eq. (5d): The inverse of the price level is the value of one unit of nominal money in units of goods it buys, so (5d) implies that the interest rate on that value is zero; however, that is a beginning-of-period nominal interest rate, as opposed to the end-of-period rate defined below in (7a) [and (14)] which is not necessarily equal to zero.

The rates of return on the two assets, money and shares, may be compared using (4a)–(4c) yielding, see Townsend (1987),

$$U'(y) \mathrm{E}[\{[1/(Q_t/P_t)](Q_{t+1}/P_{t+1} + y) - P_t/P_{t+1}\}|\cdot]$$

[5]This is a consequence of the well-known result that the stochastic growth problem can be solved independently of the asset price problem, see, e.g., Brock (1982). Eq. (5g) above is the well-behaved (no bubbles) solution for the stock price.

$$=[1/(Q_t/P_t)]\mathrm{E}[(Q_{t+1}/P_{t+1}+y)\mu_{t+1}|\cdot]. \tag{6}$$

Additionally, we may price any asset of any maturity as shown in Lucas (1982, 1984). In the case of a nominal claim of maturity k, for $k=1,2,3,\ldots$, obtained in the asset market at the end of period t, which pays one unit of cash delivered in the asset market at the end of period $t+k$ (recall fig. 1), its own end of period interest rate, i_t, is given by

$$\beta^k\mathrm{E}[\lambda_{t+k}/P_{t+k}|\cdot]=[1/(1+i_t)](\lambda_t/P_t). \tag{7a}$$

The interest on an equivalent real claim, r_t, is simply

$$\beta^k\mathrm{E}[\lambda_{t+k}|\cdot]=[1/(1+r_t)]\lambda_t. \tag{7b}$$

4. Stochastic structure

The basic idea here is to present a stochastic structure which replaces the Giovannini (1989) assumption that the liquidity constraint binds each period, or alternatively that the solution is always (5a)–(5c), with the assumption that each period agents receive a signal γ_t (an element of the set $\{0,1\}$) affecting the probability that the constraint will bind in the following period.[6] In turn, I have to restrict the distribution function of the state, $H(\omega',\omega)$, in the following manner:

$$\begin{aligned}H(\omega_{t+1},\phi_t)&=\Pr(\omega'_{t+1}\leqq\omega_{t+1}\,|\,\phi'_t=\phi_t)\\&=\alpha_t[\gamma_tH_{0g}(\omega_{t+1})+(1-\gamma_t)H_{0b}(\omega_{t+1})]\\&\quad+(1-\alpha_t)[\gamma_tH_{1g}(\omega_{t+1})+(1-\gamma_t)H_{1b}(\omega_{t+1})],\end{aligned} \tag{8a}$$

with

$$\phi_t=(\alpha_t,\gamma_t), \tag{8b}$$

$$\Pr(\alpha_{t+1}=1)+\Pr(\alpha_{t+1}=0)=1, \tag{8c}$$

$$\Pr(\gamma_{t+1}=1)+\Pr(\gamma_{t+1}=0)=1, \tag{8d}$$

$$\omega_t \text{ and } \alpha_t \text{ iid and stochastically independent}, \tag{8e}$$

$$\omega_t \text{ and } \gamma_t \text{ iid and stochastically independent}, \tag{8f}$$

[6] In essence, I am replacing the two distributions that Giovannini (1989) used by four distributions to be defined below.

$$\alpha_t \text{ and } \gamma_t \text{ iid and stochastically independent,} \tag{8g}$$

$$\int \omega_{t+1}\, \mathrm{d}H_{0g}(\omega_{t+1}) = \int \omega_{t+1}\, \mathrm{d}H_{1g}(\omega_{t+1}) = \mathrm{E}(\omega_g), \tag{8h}$$

$$\mathrm{d}H_{1g}(\omega_{t+1}) = \mathrm{MPS}(\omega_{t+1}) + \mathrm{d}H_{0g}(\omega_{t+1}), \tag{8i}$$

$$\int \omega_{t+1}\, \mathrm{d}H_{0b}(\omega_{t+1}) = \int \omega_{t+1}\, \mathrm{d}H_{1b}(\omega_{t+1}) = \mathrm{E}(\omega_b), \tag{8j}$$

$$\mathrm{d}H_{1b}(\omega_{t+1}) = \mathrm{MPS}(\omega_{t+1}) + \mathrm{d}H_{0b}(\omega_{t+1}), \tag{8k}$$

where $\mathrm{MPS}(\omega_{t+1})$ is a mean preserving spread of the distribution in the sense of Rothschild and Stiglitz (1970). The transition probabilities governing ω are given by one of four distributions, depending on draws of α_t and γ_t which are independent zero–one random variables. If $\gamma_t = 1$, the distribution of the state is bounded by $h_{t-1}(M_0) \leqq (y/\beta \mathrm{E}[1/P_{t+1} | \cdot])$ almost surely (a.s.) for all possible realizations of $\mathrm{E}[\cdot | \cdot]$, and is given by $\alpha_t H_{0g} + (1-\alpha_t) H_{1g}$ such that H_{1g} is a MPS of H_{0g} according to (8h)–(8i). If $\gamma_t = 0$, the distribution of the state is bounded by $h_{t-1}(M_0) > (y/\beta \mathrm{E}[1/P_{t+1} | \cdot])$ a.s. for all possible realizations of $\mathrm{E}[\cdot | \cdot]$, and is given by $\alpha_t H_{0b} + (1-\alpha_t) H_{1b}$ such that H_{1b} is a MPS of H_{0b} according to (8j)–(8k). Whether α_t is equal to zero or one then basically determines one of the four functions H as the distribution function. Therefore, given the equilibrium (5), the stochastic structure (8) generates a probabilistic model which signals to the agent if the cash-in-advance constraint is binding or not next period and the degree of risk of the drawing of the state reflecting the ordering 'low' and 'high' risk. Note that the distribution function of ω_{t+1} is independent of the realization of the current state, ω_t, and the current realization may be interpreted as a temporary disturbance which does not change the probability distribution of the future states as in Svensson (1985a).[7]

5. The role of monetary growth innovations

The solution in (5) presents random variables that are functions of expected values of next period's random variables which, in turn, are functions of the current innovations vector ϕ_t. Expectations, at time t, of the random variables for time $t+1, 2, 3, \ldots$ can be thought of as having a probability distribution induced by the probability of the associated elements

[7]An extensive survey of the empirical literature which models time-varying moments is found in Bollerslev et al. (1990). Krugman et al. (1985) is a theoretical model that explores a probabilistic structure for the cash-in-advance constraint in a model which allows individual uncertainty, but is deterministic in the aggregate.

of $\phi_{t,t+1,t+2,\ldots}$, respectively. The strategy of analysis is to consider, in case 1, the general solution when $\gamma_t = 1$ a.s. and, in that state, to analyze the effects of increased future monetary volatility on the endogenous prices, i.e., $\alpha_t = 1$ versus $\alpha_t = 0$. Then, case 2 is when $\gamma_t = 0$ a.s. and in this alternative state we analyze the effects of increased future monetary volatility on the endogenous prices. Table 1 summarizes the results of cases 1 and 2.

5.1. *Case 1:* $\gamma_t = 1$ *a.s.* $\rightarrow \mu_{t+1} > 0$

The innovations to the future rate of money growth signal that, for any realization of ω_{t+1}, the cash-in-advance constraint will be binding a.s. in period $t+1$, $(M_{t+1}/P_{t+1}) = y$.[8] What is the effect of changes in future money growth volatility on the endogenous prices? The solution for the current purchasing power of money is (5a), and its expected value for $t+1$ is $\mathrm{E}[1/P_{t+1}|\cdot, \gamma_t = 1] = [y/\omega_t h_{t-1}(M_0)]$, so future monetary volatility does not affect $1/P_t$ or $\mathrm{E}[1/P_{t+1}|\cdot]$. Alternatively, $\alpha_t = 1$ versus $\alpha_t = 0$ has no effect. In fact, the next period purchasing power of money may be seen as the end of current period purchasing power of money, therefore a function of the current state. This reflects the assumption that the monetary transfer (tax) can only be used for consumption in the next period's goods market. In turn, there is an effect on the expected value of the purchasing power of money for period $t+2$, conditional on ϕ_t. This is given by

$$
\begin{aligned}
&\mathrm{E}[1/P_{t+2}|\alpha_t, \gamma_t = 1] \\
&\quad = \Pr(\gamma_{t+1} = 1)[y/h_{t-1}(M_0)\omega_t]\mathrm{E}[1/\omega_{t+1}|\alpha_{t+1}, \alpha_t, \gamma_{t+1} = 1, \gamma_t = 1] \\
&\qquad + \Pr(\gamma_{t+1} = 0)\beta\{\Pr(\gamma_{t+2} = 1)[y/h_{t-1}(M_0)\omega_t] \\
&\qquad \times \mathrm{E}[1/\omega_{t+1}|\alpha_{t+2}, \alpha_{t+1}, \alpha_t, \gamma_{t+2} = 1, \gamma_{t+1} = 0, \gamma_t = 1] \\
&\qquad \times \mathrm{E}[1/\omega_{t+2}|\alpha_{t+2}, \alpha_{t+1}, \alpha_t, \gamma_{t+2} = 1, \gamma_{t+1} = 0, \gamma_t = 1] \\
&\qquad + \Pr(\gamma_{t+2} = 0)\beta\{\Pr(\gamma_{t+3} = 1)\ldots \qquad (9)
\end{aligned}
$$

The result of eq. (9) reflects the fact that there is always some positive probability that the future cash-in-advance constraint will be binding. Here, I make a simplifying assumption that $\mathrm{E}[1/P_{t+3,4,5\ldots}|\cdot]$ is constant and equal

[8]Note that, without loss of generality, the current state is assumed to be consistent with $\mu_t > 0$. In case 1, monetary velocity is expected to be constant equal to one for $t+1$.

Table 1

Effects of future monetary growth volatilty on endogenous prices ($\alpha_t = 1$ versus $\alpha_t = 0$).[a]

	Case 1: $\gamma_t = 1$ a.s. $\rightarrow \mu_{t+1} > 0$		Case 2: $\gamma_t = 0$ a.s. $\rightarrow \mu_{t+1} = 0$	
$E[1/P_{t+1} \mid \phi_t]$	(0)		(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[1/P_{t+2} \mid \phi_t]$	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
λ_t	(0)		(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[\lambda_{t+1} \mid \phi_t]$	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(0)	
μ_t	(0)		(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[\mu_{t+1} \mid \phi_t]$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(0)	(=0)
λ_t/P_t	(0)		(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[\lambda_{t+1}/P_{t+1} \mid \phi_t]$	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[\lambda_{t+2}/P_{t+2} \mid \phi_t]$	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
μ_t/P_t	(0)		(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[\mu_{t+1}/P_{t+1} \mid \phi_t]$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(0)	(=0)
Q_t/P_t	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 1$)	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
Rate of return dominance (RRD)	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	No. RRD	
i_t $(k = 1)$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(0)	i_t $(k = 1) = 0$
i_t $(k = 2)$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(0)	
i_t $(k = 3, 4, \ldots)$	(0)		(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[1/(1 + i_{t+1}) \mid \phi_t]$ $(k = 2, 3, \ldots)$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
r_t $(k = 1)$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
r_t $(k = 2, 3, \ldots)$	(0)		(+) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)
$E[1/(1 + r_{t+1}) \mid \phi_t]$ $(k = 1, 2, \ldots)$	(−) (0)	w/Pr($\gamma_{t+1} = 1$) w/Pr($\gamma_{t+1} = 0$)	(0)	

[a] (+) → increase, (−) → decrease, (0) → unchanged, w/Pr($\gamma_{t+1} = 1, 0$) → with probability γ_{t+1} equals one or zero.

to the unconditional expectation $\mathrm{E}[1/P]$.[9] With that in mind, the second term on the right-hand side of (9) becomes $\Pr(\gamma_{t+1}=0)\beta\mathrm{E}[1/P]$. The first conditional expectation on the right-hand side of (9) is a convex function of ω_{t+1} such that we may establish that

$$\mathrm{E}[1/\omega_{t+1}|\alpha_t=1]<\mathrm{E}[1/\omega_{t+1}|\alpha_t=0], \tag{10}$$

conditionally and unconditionally on γ_t. The key aspect of (9), is that there is a greater dispersion of outcomes so that the new information about $t+2$, arriving in $t+1$, has a greater value. In turn, an increase in future money growth volatility increases, by (10), the expected purchasing power of money with $\Pr(\gamma_{t+1}=1)$. The expected purchasing power of money is unchanged with $\Pr(\gamma_{t+1}=0)$.[10] We can associated this result with smoothing of asset demands. Even if the current innovations signal a draw from a distribution which implies a binding cash-in-advance constraint for $t+1$, $\gamma_t=1$ a.s. say, a high probability of opposed innovations for $t+2$, high $\Pr(\gamma_{t+1}=0)$ say, smooths expected future price fluctuations and asset demands in response to higher money growth volatility for period $t+1$. In other words, asset demand is unchanged because it is optimal to wait for the arrival of new information. This is also consistent with the concept of greater dispersion being associated with the lengthening of the optimal search time.

The current marginal utility of real wealth, λ_t, is unchanged by nominal volatility as may be seen from (5b). Indeed, it may be expressed as $\lambda_t=\beta U'(y)/\omega_t$ which shows that the intertemporal valuation of the monetary asset in terms of consumption is not affected by future monetary volatility. The expectation of the marginal utility of real wealth for $t+1$, conditional on ϕ_t, is given by

$$\mathrm{E}[\lambda_{t+1}|\alpha_t,\gamma_t=1]=[\beta U'(y)\omega_t h_{t-1}(M_0)/y]\,\mathrm{E}[1/P_{t+2}|\alpha_t,\gamma_t=1]. \tag{11a}$$

By (10), $\mathrm{E}[\lambda_{t+1}|\cdot]$ increases with $\Pr(\gamma_{t+1}=1)$ and is unchanged with

[9] This assumption basically gives me analytical tractability. Note that in (9), the terms $\mathrm{E}[1/P_{t+3,4,\ldots}|\cdot]$ are disperse according to the elements of $\phi_{t+2,3,4\ldots}$ and so on. Results would be only slightly altered in the sense that the additional probabilities for $\phi_{t+2,3,4\ldots}$ would appear as multiplied by the probabilities for ϕ_{t+1} and so on for all future states, so that the total effects would become smaller as t becomes larger. Therefore, the assumption above may be a suitable first order approximation.

[10] If $\Pr(\gamma_{t+1}=1)=1$, then we obtain the usual result obtained by Giovannini (1989) who analyzed the case where the cash-in-advance constraint binds with probability one for all states. See also Stulz (1986, p. 341) for results which draw on an inequality as (10).

$\Pr(\gamma_{t+1}=0)$. The effect on the expected marginal utility of real wealth for $t+1$ is due to a wealth effect that arises from the future higher monetary volatility. The expectation for $t+2$ is

$$\begin{aligned}\mathrm{E}[\lambda_{t+2}|\alpha_t,\gamma_t=1]&=\Pr(\gamma_{t+1}=1)[\beta\omega_t h_{t-1}(M_0)U'(y)/y]\\&\quad\times\mathrm{E}[\omega_{t+1}|\alpha_{t+1},\alpha_t,\gamma_{t+1}=1,\gamma_t=1]\,\mathrm{E}[1/P]\\&\quad+\Pr(\gamma_{t+1}=0)U'(y),\end{aligned}\tag{11b}$$

and since the first expectation on the right-hand side is a linear function of ω_{t+1}, it is unaffected by changes in monetary volatility conditionally and unconditionally. The effects on λ_t are fully transmitted to μ_t by (5c): The current value μ_t, which may be expressed as $\mu_t=(1-\beta/\omega_t)U'(y)$, is unchanged and $\mathrm{E}[\mu_{t+1}|\cdot]$ decreases with $\Pr(\gamma_{t+1}=1)$ and is unchanged with $\Pr(\gamma_{t+1}=0)$ given the higher future monetary volatility. The reason for the effect on $\mathrm{E}[\mu_{t+1}|\cdot]$ is because a lower relative demand for money arises with the expected future monetary volatility and a lower expected return for the liquidity services of money is verified.

The effect on the marginal utility of nominal wealth, λ_t/P_t, is obtained by (5b) and (5a). The current value of λ_t/P_t is unchanged. The expected value for $t+1$, conditional on ϕ_t, is given by

$$\mathrm{E}[\lambda_{t+1}/P_{t+1}|\alpha_t,\gamma_t=1]=\beta U'(y)\mathrm{E}[1/P_{t+2}|\alpha_t,\gamma_t=1],\tag{12a}$$

so that, by (9), $\mathrm{E}[\lambda_{t+1}/P_{t+1}|\cdot]$ increases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged with $\Pr(\gamma_{t+1}=0)$. Similarly, the expected value for $t+1$, conditional on ϕ_t, of the liquidity value of nominal money is affected such that $\mathrm{E}[\lambda_{t+1}/P_{t+1}|\cdot]$ decreases with $\Pr(\gamma_{t+1}=1)$ and is unchanged with $\Pr(\gamma_{t+1}=0)$. The expected value for $t+2$ of the marginal utility of nominal wealth, $\mathrm{E}[\lambda_{t+2}/P_{t+2}|\phi_t]$ is given by

$$\begin{aligned}\mathrm{E}[\lambda_{t+2}/P_{t+2}|\alpha_t,\gamma_t=1]&=\Pr(\gamma_{t+1}=1)\\&\quad\times\{\beta U'(y)\mathrm{E}[\omega_{t+1}|\alpha_{t+1},\alpha_t,\gamma_{t+1}=1,\gamma_t=1]\\&\quad\times\mathrm{E}[1/\omega_{t+1}|\alpha_{t+1},\alpha_t,\gamma_{t+1}=1,\gamma_t=1]\,\mathrm{E}[1/P]\}\\&\quad+\Pr(\gamma_{t+1}=0)\{\beta U'(y)\mathrm{E}[1/P]\},\end{aligned}\tag{12b}$$

and, by (10), it increases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged with $\Pr(\gamma_{t+1}=0)$. Note that, if $\mathrm{E}[\lambda_{t+2}/P_{t+2}|\cdot]$ increases, it does increase by exactly the same amount as $\mathrm{E}[\lambda_{t+1}/P_{t+1}|\cdot]$ from (12a). The expected value of the liquidity value of nominal money for $t+2$, conditional on ϕ_t, is given by

$$\begin{aligned}E[\mu_{t+2}/P_{t+2}|\alpha_t,\gamma_t=1]&=\Pr(\gamma_{t+1}=1)\\&\times\{U'(y)E[1/P_{t+2}|\alpha_{t+1},\alpha_t,\gamma_{t+1}=1,\gamma_t=1]\\&-E[\lambda_{t+2}|P_{t+1}|\alpha_{t+1},\alpha_t,\gamma_{t+1}=1,\gamma_t=1]\}\\&+\Pr(\gamma_{t+1}=0)(0)\end{aligned}\qquad(13)$$

and, by (9) and (12b), $E[\mu_{t+2}/P_{t+2}|\cdot]$ is unchanged. This result is established since with $\Pr(\gamma_{t+1}=1)$ the effects on $E[1/P_{t+2}|\cdot]$ and $E[\lambda_{t+2}/P_{t+1}|\cdot]$ cancel out and with $\Pr(\gamma_{t+1}=0)$ it is trivially unchanged.

Stock prices and the real interest rate) are a function of λ_t and $E[\lambda_{t+1,2,\dots}|\cdot]$ by (5g) [and (7b)]. The current stock price may or may not respond immediately to increases in future money volatility because of the effect on $E[\lambda_{t+1}|\cdot]$. By (11a), $E[\lambda_{t+1}|\cdot]$ increases with $\Pr(\gamma_{t+1}=1)$ so that the current stock price also increases with $\Pr(\gamma_{t+1}=1)$. However, the current stock price is unchanged with $\Pr(\gamma_{t+1}=0)$. The announcement of higher future money growth volatility may increase the current demand for stocks because the expected liquidity value of real balances for $t+1$ is expected to decrease by (11a). Alternatively, higher future money growth volatility increases the current demand for stocks relative to real money balances.

The effect of increased monetary volatility on asset valuation is seen from (6). The gap in rates of return between money and shares decreases with $\Pr(\gamma_{t+1}=1)$ and is unchanged with $\Pr(\gamma_{t+1}=0)$. This follows from the effect on the expected liquidity value of money for $t+1$, $E[\mu_{t+1}|\cdot]$, and on the current stock price, Q_t/P_t, both in the same direction with respect to the gap in rates of return. Again, this is because the increase in demand for stocks, relative to money, decreases the relative stock return. However, with $\Pr(\gamma_{t+1}=0)$ relative asset demands are unchanged and the gap in rates of return is also unchanged.[11]

The results on the term structure of interest rates are obtained from (7) all conditional on ϕ_t. The nominal interest rate for the one-period bond, $k=1$, decreases with $\Pr(\gamma_{t+1}=1)$, and stays unchanged with $\Pr(\gamma_{t+1}=0)$, by the effect on $E[\lambda_{t+1}/P_{t+1}|\cdot]$ as in (12a). In other words, the nominal wealth value of the future marginal gain is expected to increase so that, other things equal, the current nominal return must fall. For $k=2$ (the two-period bond) the nominal interest rate decreases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged

[11]The results above are the distinctive feature of the cash-in-advance setting that variations in the constraint multiplier affect the valuation of the other assets, see, e.g., Townsend (1987).

with $\Pr(\gamma_{t+1}=0)$ from the effect of $E[\lambda_{t+2}/P_{t+2}|\cdot]$ as seen in (12b). The conditional expectation of the nominal rate for $t+1$ will depend on $E[\lambda_{t+1}/P_{t+1}|\cdot]$ for all maturities $k=1,2,3,\ldots$, and $E[\lambda_{t+k}/P_{t+k}|\cdot]$ for each maturity $k=2,3,\ldots$. So, the expected rate for the one-period bond stays unchanged, because if $\Pr(\gamma_{t+1}=1)$, the effects on $E[\lambda_{t+1}/P_{t+1}|\cdot]$ and $E[\lambda_{t+2}/P_{t+2}|\cdot]$ are identical and cancel out and if $\Pr(\gamma_{t+1}=0)$ the values are unchanged. The expected rate for all other maturities increases, with $\Pr(\gamma_{t+1}=1)$, and stays unchanged with $\Pr(\gamma_{t+1}=0)$ by the effect of $E[\lambda_{t+1}/P_{t+1}|\cdot]$. The results for the real interest rate are similarly obtained with changes dictated by the term $E[\lambda_{t+1}|\cdot]$ from (11). The one-period real rate decreases with $\Pr(\gamma_{t+1}=1)$, while for $k=2,3,\ldots$, the term structure is unaffected. The expected real rate for $t+1$ increases with $\Pr(\gamma_{t+1}=1)$ for all maturities.

In summary, when the current monetary innovations signal that the realization of the future rate of money growth is consistent with a binding cash-in-advance constraint in period $t+1$, increases in future monetary volatility *may have* an effect on asset holdings, *increasing* the relative demand for stocks and bonds with a portfolio adjustment away from money balances. However, the effects depend critically on the arrival of new information for $t+1$. In fact, there is a probability, $\Pr(\gamma_{t+1}=0)$, that future increased monetary volatility has no impact on asset holdings, as well as in any of the nominal and real variables in the economy. In this case, it is optimal to wait for the arrival of new information. Alternatively, increases in future monetary risk affect the endogenous variables only if the future state is expected to be drawn from a specific distribution. Given a current state, speculations about future changes only have an effect if they belong to a certain specific class. With two different classes of innovations, only the ones consistent with $\gamma_{t+1}=1$ a.s. have an effect. More importantly, they do not cancel out.[12]

5.2. *Case 2:* $\gamma_t=0$ *a.s.* $\rightarrow \mu_{t+1}=0$

In this case, the innovations about the future rate of money growth signals that for any realization of ω_{t+1}, the cash-in-advance constraint in period $t+1$ will not be binding and $(M_{t+1}/P_{t+1})>y$.[13] Again, the solution for the current purchasing power of money is (5a), and (5b)–(5c) may be expressed alternatively, given that in this case $E[1/P_{t+1}|\cdot,\gamma_t=0]=\beta E[1/P_{t+2}|\cdot,\gamma_t=0]$, yielding

$$\lambda_t=[\beta^2 U'(y)h_{t-1}(M_0)/y]E[1/P_{t+2}|\phi_t], \tag{5b'}$$

[12] This is equivalent to the 'bad news principle' in the irreversibility literature of investment as in Bernanke (1983) where only downside uncertainty matters.

[13] Monetary velocity is expected to be variable and less than one for $t+1$.

$$\mu_t = U'(y) - [\beta^2 U'(y) h_{t-1}(M_0)/y] \mathrm{E}[1/P_{t+2} | \phi_t]. \tag{5c'}$$

Interestingly, $\mathrm{E}[1/P_{t+2}|\cdot]$ is given by (9) conditionally and unconditionally on γ_t. Therefore, in this case, an increase in future money growth volatility increases the expected purchasing power of money for $t+1$ with $\Pr(\gamma_{t+1}=1)$ and leaves it unchanged with $\Pr(\gamma_{t+1}=0)$. This is because the current announcement signals that the expected marginal liquidity value of money for $t+1$ is zero, but, with $\Pr(\gamma_{t+1}=1)$, it is expected to increase for $t+2$, therefore providing an infinite gain for the marginal money holdings in $t+1$. In turn, agents are expected to increase their money demand for $t+1$ bidding up the expected price of money for $t+1$. The outcome is dispersed by the $\Pr(\gamma_{t+1}=0)$ which would invalidate the expected increase in the marginal liquidity value of money for $t+2$ and would leave the money demand for $t+1$ unchanged. Note also that the increase in $\mathrm{E}[1/P_{t+1}|\cdot]$ is exactly equal to the increase in $\mathrm{E}[1/P_{t+2}|\cdot]$.

In this case, the intertemporal valuation of money in terms of consumption, as in (5c′), will be affected. First, the current marginal utility of real wealth, λ_t, increases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged with $\Pr(\gamma_{t+1}=0)$ as may be seen from (5b′). The expectation for $t+1$ is, by (5e), $\mathrm{E}[\lambda_{t+1}|\cdot,\gamma_t=0]=U'(y)=\mathrm{E}[\lambda]$ constant and unchanged as well as the expectation for $t+2$ given by (11b). The current liquidity value of real money, μ_t, decreases with $\Pr(\gamma_{t+1}=1)$ because of the effect on λ_t. Its expected values for $t+1,2,3,\ldots$ are all unchanged. The result here is that a marginal increase in current money holdings provides a higher discounted value of future consumption and a lower gain derived from relaxing the current liquidity constraint as measured by μ_t. In particular, note that $\mathrm{E}[\mu_{t+1}|\gamma_t=0]=0$ independently of α_t.

It is the asset valuation in (6) and the nominal one-period interest rate in (7) that are critically affected by $\gamma_t=0$ a.s. It implies that the expected liquidity value of money for $t+1$ equals zero so (6) equals zero and the rates of return between money and shares are equated. This also implies that the current nominal one-period interest rate is zero since it may be expressed as

$$i_t = \mathrm{E}[\mu_{t+1}/P_{t+1} | \gamma_t = 0] / \mathrm{E}[\lambda_{t+1}/P_{t+1} | \cdot] = 0, \tag{14}$$

independently of α_t. The announcement means that in period $t+1$, money will be drawn from a distribution associated with low values of the state such that no other asset can dominate money in rate of return, which may be seen by (6) equal to zero. In turn, the end-of-period current nominal interest rate must equal zero too. Note that this result is the original result in Lucas (1982) with a different sequence of markets, i.e., the asset market is open after the current state is known and before the goods market open (see fig. 1). Intuitively, in our case, the announcement of future zero liquidity value of money gives the agent additional information about the future state *before* the current asset market closes so that the agent *can* adjust money holdings before going to the future goods market.[14]

The current marginal utility of nominal wealth, λ_t/P_t, is obtained from (5b′) and (5a)

$$\lambda_t/P_t=\beta^2 U'(y)\mathrm{E}[1/P_{t+2}|\alpha_t,\gamma_t=0], \tag{15a}$$

and, by (9), it increases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged with $\Pr(\gamma_{t+1}=0)$. Identical result holds for the expected value for $t+1$, given by [from (5d)–(5e)]

$$\mathrm{E}[\lambda_{t+1}/P_t|\alpha_t,\gamma_t=0]=\beta U'(y)\mathrm{E}[1/P_{t+2}|\alpha_t,\gamma_t=0]. \tag{15b}$$

It increases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged with $\Pr(\gamma_{t+1}=0)$. The expected value for $t+2$, $\mathrm{E}[\lambda_{t+2}/P_{t+2}|\cdot]$, is given by (12b) above so that it increases with $\Pr(\gamma_{t+1}=1)$ and stays unchanged with $\Pr(\gamma_{t+1}=0)$. These results are reflected in μ_t/P_t, the liquidity value of nominal money. Its current value decreases with $\Pr(\gamma_{t+1}=1)$ as a result of (15a). $\mathrm{E}[\mu_{t+1}/P_{t+1}|\cdot]=0$ is unchanged, and $\mathrm{E}[\mu_{t+2}/P_{t+2}|\cdot]$ is also unchanged by the same rationale as in (13).

Also in this case, the current stock price may or may not respond immediately to changes in future monetary volatility. It is a function of the current marginal utility of real wealth, λ_t. In turn, being inversely related to λ_t, it may decrease with $\Pr(\gamma_{t+1}=1)$ or it may stay unchanged with $\Pr(\gamma_{t+1}=0)$. In fact, the result here is diametric to the case when $\gamma_t=1$. In this case, the stock price may decrease, with $\Pr(\gamma_{t+1}=1)$, because of the anticipated gain on money holdings for $t+1$ and $t+2$. Agents increase their money holdings in the asset market at the end of period t relative to stocks, therefore decreasing the current stock price. If the gain is not expected to occur, that is with $\Pr(\gamma_{t+1}=0)$, then expected future monetary volatility has no effect on the current relative demand for stocks.

Finally, the effects on the interest rates and the term structure are now easily obtained. The nominal rate for $k=2$ is a function of λ_t/P_t and $\mathrm{E}[\lambda_{t+2}/P_{t+2}|\cdot]$. By (12b) and (15a) they respond identically to future monetary volatility so that the nominal rate for $k=2$ is unchanged. For all other maturities, $k=3,4,\ldots$, the current nominal rate increases with $\Pr(\gamma_{t+1}=1)$ or it stays unchanged with $\Pr(\gamma_{t+1}=0)$ by (15a). The expected value of the nominal rate for $t+1$ for bonds of one-period maturity is unchanged because the effects on $\mathrm{E}[\lambda_{t+1}/P_{t+1}|\cdot]$ and $\mathrm{E}[\lambda_{t+2}/P_{t+2}|\cdot]$ cancel

[14] If states were completely independent, $H(\omega_{t+1},\phi_t)=H(\omega_{t+1})$ as in Svensson (1985a), then this result would not hold.

out. The expected value for all other maturities, $k=2,3,\ldots$, increases because of the effect of $\mathrm{E}[\lambda_{t+1}/P_{t+1}|\cdot]$. The real interest rate is a function of λ_t and $\mathrm{E}[\lambda_{t+1,2,\ldots}|\cdot]$ so that the current real rate, r_t, increases with $\Pr(\gamma_{t+1}=1)$ for all maturities as a consequence of the effect of future monetary volatility on λ_t. In this case, agents decrease the relative demand for real bonds and again we note that the result is diametric to the case where $\gamma_t=1$ for the same reason of the stock price. The $\mathrm{E}[\lambda_{t+1,2,\ldots}|\cdot]$ are all unaffected by monetary volatility and the expected value of the real rate for $t+1$ is unchanged for all maturities.

In summary, when the current monetary innovations signal that the realization of the future money growth will be consistent with a nonbinding cash-in-advance constraint in period $t+1$, increases in future monetary volatility *may have* an effect on asset holdings, *decreasing* the relative demand for stocks and bonds with a portfolio adjustment into money balances and away from stocks and bonds. In fact, it is a store-of-value demand for money that arises in this case. Similarly, the effects depend critically on the arrival of new information for $t+1$, so that, with $\Pr(\gamma_{t+1}=0)$, future increased monetary volatility has no impact in asset holdings, as well as in any of the nominal and real variables in the economy.

6. An interpretation of the results

The interpretation of the cash-in-advance economy, in general, and, of the framework above, in particular, is that the cash-in-advance constraint (1c) introduces an element of irreversibility to the representative agent's optimal choice between consumption, money, stocks, and bonds. A decision on the amount of assets to carry over to period $t+1$ is made at time t, with information as of time t. However, once that decision is made, the future choice of consumption for time $t+1$, with new information as of $t+1$, is constrained by the previous decision as of time t. This is because money provides full liquidity while stocks and bonds do not. Alternatively, if at time t the demand for money falls and the demand for shares increases, the agent has a smaller option value in terms of the amount he may consume at $t+1$. Similarly, if at time t the expected demand for money for period $t+1$ falls (and the expected demand for shares increases) the option value in terms of consumption is expected to be smaller for $t+2$. What is a measure of that option value in our monetary framework? Obviously, it is the liquidity value of money μ_t. This explains our basic results: In case 1, higher expected money growth volatility leads to an increase in the relative demand for stocks and bonds which is reflected in a lower option value for consumption in $t+1$ as measured by $\mathrm{E}[\mu_{t+1}|\cdot]$ (and $\mathrm{E}[\mu_{t+1}/P_{t+1}|\cdot]$); in case 2, the results are critically different because $\gamma_t=0$ a.s. imples that the option (liquidity) value for $t+1$ disappears and money is as attractive an asset as all others in terms of rate of return. Therefore, the current liquidity value falls (the storage value of money increases) and the demands for the other assets fall. Indeed, the cash-in-advance constraint and the timing of events in fig. 1 lead to a general combined transactions, precautionary, and store-of-value demand for money. However, the new insight here, which may be seen in table 1, is that most of the result in cases 1 and 2 depend on the arrival of new information for $t+1$, such that it might be optimal to wait for the arrival of the new information while leaving asset demands unchanged, say when there is a high $\Pr(\gamma_{t+1}=0)$. Notice that these results are completely independent of preferences (the curvature of the utility function) and are associated with the opportunity cost of postponing a commitment in order to wait for the arrival of new information.[15]

7. Summary and conclusions

We have presented a stationary stochastic rational expectations model with time-varying distributions of the state. We analyzed the role of exogenous monetary shocks in the form of innovations in conditional variances. Specifically, we have shown that when agents receive a signal that the future cash-in-advance constraint will bind almost surely, an increase in the conditional variance of the money growth process increases the demand for stocks and bonds relative to money with an increase in the price of stocks and bonds. However, if agents receive a signal that the cash-in-advance constraint will not bind almost surely, the result is reversed. There is an increase in the demand for money relative to stocks and bonds with a consequent decrease in stocks and bonds prices. Given the sequence of markets and the timing of information, these effects are dispersed with $\Pr(\gamma_{t+1}=0)$ in which case it is optimal to wait for the arrival of new information in the next period such that asset demands are relatively unchanged. They key to understand our basic result is to note that when $\gamma_t=1$, and increase in the money growth volatility does not affect the current marginal valuation of real wealth, but it increases its expected value for $t+1$. However, when $\gamma_t=0$, an increase in the money growth volatility increases the current marginal utility of real wealth while its expected value for $t+1$ is unchanged. In turn, the resulting effect on stock and bond prices is reversed in each case.

Even though we have restricted the distributions and the probabilistic structure of some of the future states, our exercise shows that the class of models used in this paper may answer standard macro questions in a rich

[15] In fact, the option value derived from the irreversibility effect holds even under risk neutrality. See, e.g., Cukierman (1980) and Bernanke (1983). Marschak (1949) is an original contribution on the liquidity of an asset under different degrees of available information. See also Jones and Ostroy (1984) on some alternative applications.

general equilibrium stochastic environment, which is also important from a methodological point of view.[16] Two natural extensions to the present analysis are: To examine announcements of future real shocks and to examine the role of money growth volatility with variable velocity empirically.[17]

[16]Authors such as Lucas (1982), Svensson (1985b), and Stockman and Svensson (1987) have used this framework for open economies.

[17]Hodrick (1989) is an attempt to examine the role of changes in the conditional variance of the exogenous stochastic driving processes in an open economy, but with the assumption that the cash-in-advance constraints binds every period.

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