# OPTIMAL FISCAL POLICY IN A DYNAMIC MULTICOUNTRY WORLD WITH PUBLIC SERVICES

MARCELO BIANCONI\* Tufts University, USA
THEODORE PALIVOS Louisiana State University, USA

Abstract. The authors construct a dynamic one-good multicountry growth model with productive government spending and perfect capital mobility to study fiscal interdependence among countries. In the case of a source-based tax, it is found that there is no strategic interaction and that the equilibrium spending/tax mix coincides with the optimal one. In the case of a residence-based tax system, however, there is strategic interaction across countries and, under noncooperation, countries tend to spend and tax over and above the optimal Pigouvian level.

## 1. INTRODUCTION

This paper develops a one-good multicountry growth model with perfect capital mobility to examine a problem of taxation and government spending choice and its international repercussions. Economic growth in each nation is driven by productive government services, which are financed by either a source- or a residence-based capital income tax. We characterize the arbitrage free equilibrium with endogenous capital flows for the multicountry world.

A key contribution of this paper relates to the extension of the production efficiency theorem of Diamond and Mirrless (1971) to the open economy by Razin and Sadka (1991). The latter authors show that a residence-based taxation system preserves the Diamond and Mirrless production efficiency whereas a source-based system does not. The reason is that in the residence-based case the before-tax real returns are equated across countries but in the source-based case it is the after-tax real return that is equated. We show in this paper that in the case of "large" countries and productive public services this result is reversed. The logic is simple: in the presence of productive public services an optimal tax can fully internalize the international spillover effects in the source-based system, but cannot do the same in the residence-based system.

The international spillovers we examine derive from the effect of tax and government spending on the equilibrium world interest rate. In a closed economy, the optimal fiscal policy is derived by equating the marginal cost of raising funds with the marginal benefit of government spending. Within a multicountry world, we show that the same optimal spending and tax mix pertains also to the case of a source-based tax and that therefore the

<sup>\*</sup> Address for correspondence: Department of Economics, Tufts University, Medford, MA 02155, USA. Tel: (617) 627-2677; Fax: (617) 627-3917; E-mail: mbiancon@emerald.tufts.edu. We thank Glenn Hubbard, two anonymous referees of this journal and the editor, Kenneth Chan, for helpful comments on previous versions of the paper.

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noncooperative equilibrium is efficient, contrary to the Razin and Sadka extension of Diamond and Mirrless. Nevertheless, in the case of a residence-based tax, individual countries are unable to fully internalize the effect that their spending has on the world interest rate. Hence, the noncooperative outcome is one where the Nash equilibrium levels of taxation and government spending are above the optimal levels; i.e. countries have a tendency to overspend and overtax. Put differently, a coordination failure exists and cooperation is in general superior to noncooperation, since in the former case countries can internalize the negative spillovers.

We also find that the noncooperative equilibrium is not unique. The reason for the nonuniqueness is simple: the absence of arbitrage, which equates the before-tax interest rates, renders the simultaneous choice of optimal tax and spending indeterminate under noncooperation. However, there is a bounded set of possible noncooperative equilibria and the unique cooperative outcome is just one among the infinite elements of this set.

Our paper relates to Devereux and Mansoorian (1992), who examine the effects of policy coordination in a two-country, two-good framework with zero net asset position across countries and no direct taxes applied to international lending. One of their results is that productive government spending creates no strategic effects whereas government spending that enters the utility function does. We find that, in the case of a residence-based tax, there is strategic interaction generated by productive government spending since in our model there is a nontrivial asset position and capital income generated abroad is taxable. We relate to Chari and Kehoe (1990) as well, who are concerned with an optimal tariff problem in a multicountry, multigood world. Their focus is on the strategic effects of policies that affect the static terms of trade (relative prices). Here we have a one-good world and focus on the optimal tax/spending mix in the presence of productive public services; thus, our focus is on the strategic effects of policies that affect the intertemporal terms of trade (real interest rate). Other authors discuss some related issues in models of a small open economy. For example, Milesi-Ferreti et al. (1994) discuss the case of a small open economy without a productive role for government spending. Turnovsky (1997) also focuses on a small open economy with adjustment costs and analyzes the intertemporal path of the optimal tax rate. Finally, Ha and Sibert (1997) analyze strategic capital taxation in large open economies but without ongoing growth.<sup>2</sup>

Section 2 presents the dynamic multicountry macroeconomic structure. Section 3 presents the core of the paper: the analytics of fiscal policy under alternative tax systems (source- and residence-based) and strategic regimes

<sup>&</sup>lt;sup>1</sup> Razin and Yuen (1992) also examine the intertemporal path of the optimal tax using a two-country growth framework and find that, under residence-based taxation schemes, the tax rate should be positive in the short run and zero in the long run. In our case, there are no transitional dynamics and the optimal tax/spending policy is invariant.

 $<sup>^2</sup>$  More specifically, Ha and Siebert (1997) compute time-consistent optimal policies whereas in our framework time consistency is always guaranteed.

(cooperative and noncooperative). Section 4 concludes with some suggestions for future work.

#### 2. DYNAMIC MULTICOUNTRY MACROECONOMIC STRUCTURE

The framework is founded on the optimization principles of a representative agent. Each agent (country) is indexed by the integer i, an element of the finite set of countries I. There is also a simple competitive framework for firms and governments in each country  $i \in I$  and perfect capital mobility across countries. When necessary, a variable x is indexed as  $x_i^j$ , indicating that the location of x is in country  $j \in I$  and the ownership is of the resident of country  $i \in I$ . There is only one commodity which is freely traded. Throughout the paper we assume that countries are identical in every aspect except perhaps for their initial wealth,  $W_i(0)$ ,  $i \in I$ .

Endogenous growth is generated by productive government spending, which is financed by a tax on capital income.<sup>4</sup> Define the variable  $k_i^j(t)$  as the capital stock at time t which is located in country  $j \in I$  and is owned by the resident of country  $i \in I$ . Hence, it follows that the wealth of the resident of country i is given by

$$W_{i}(t) = \sum_{j=1}^{I} k_{i}^{j}(t).$$
 (1)

It is assumed that  $W_i(t) > k_i^j(t) \ge 0$  for all  $j \in I$ ,  $j \ne i$ . Let also  $\tau_i^{jm} \in [0, 1]$  denote the tax rate imposed by the government of country i on capital income which originates in country j and is owned by the resident of country m, where  $i, j, m \in I$  and either i = j (a source tax) or i = m (a residence tax). The building blocks of the model are then as follows.

## 2.1. Households

The representative household in country i faces the following optimization problem:

$$\max \int_0^\infty U(c_i(t)) \exp(-\beta t) dt \tag{2}$$

subject to the budget constraint

$$\dot{W}_{i}(t) = (1 - \tau_{i}^{ii})r_{i}(t) \left[ W_{i}(t) - \sum_{j=1}^{I,j \neq i} k_{i}^{j}(t) \right] - (1 - \tau_{i}^{ji}) \sum_{j=1}^{I,j \neq i} (1 - \tau_{j}^{ji})r_{j}(t)k_{i}^{j}(t) - c_{i}(t),$$
(3)

 $<sup>^{3}</sup>$  Throughout the paper, we use the symbol I to denote both the index set of countries as well as the number of its elements (cardinality).

<sup>&</sup>lt;sup>4</sup>We thus assume that the government in each country cannot rely on lump-sum taxes. Nevertheless, our results would be similar if lump-sum taxes were allowed. See Barro (1990) for the analysis of such a case in a closed-economy framework.

the non-negativity constraint

$$k_i^j(t) \ge 0, (4)$$

and a given level of initial wealth

$$W_i(0) = \sum_{j=1}^{I} k_i^j(0),$$

where  $c_i$  denotes per capita consumption,  $\beta > 0$  is the constant rate of time preference and  $r_i$  stands for the interest rate in country  $i \in I$ . According to (3), the net change in wealth in every period equals the difference between disposable income and consumption. Disposable income, in turn, consists of the net of taxes income raised at home (first term on the right-hand side) and the net of taxes income raised in all other countries (second term on the right-hand side). It should also be noted that, in solving the aforementioned optimization problem, agents take all government variables as given. Furthermore, throughout the paper, we assume that the utility function takes the form  $U(c) = (c^{1-\sigma} - 1)/(1-\sigma)$ , where  $\sigma$  is the inverse of the constant elasticity of substitution.

The first-order necessary conditions for a maximum are

$$(c_i(t))^{-\sigma} = \lambda_i(t), \tag{5}$$

$$0 \ge -(1 - \tau_i^{ii})r_i(t) + (1 - \tau_i^{ji})(1 - \tau_i^{ji})r_j(t), \quad k_i^j(t) \ge 0, \quad \forall j \in I, \quad j \ne i$$
 (6)

with complementary slackness

$$\dot{\lambda}_i(t) = \beta \lambda_i(t) - \lambda_i(t)(1 - \tau_i^{ii})r_i(t), \tag{7}$$

the transversality condition

$$\lim_{t \to \infty} \lambda_i(t) W_i(t) \exp(-\beta t) = 0, \tag{8}$$

and the constraint (3), where  $\lambda_i$  denotes the costate variable associated with (3). A necessary and sufficient condition for the transversality condition, equation (8), to hold as well as for the lifetime utility, given in (2), to be bounded is  $\beta + (\sigma - 1)(1 - \tau_i^{ii})r_i > 0$  (see the expression for the growth rate of wealth given in (13) below). This condition is henceforth assumed to hold.<sup>6</sup>

#### 2.2. Governments

Governments are assumed to operate under either a source- or a residence-based tax scheme. Furthermore, they balance their budget in every period.<sup>7</sup>

<sup>&</sup>lt;sup>5</sup>It can be shown that a constant elasticity of substitution as well as a constant rate of time preference are both necessary and sufficient conditions for the existence of a balanced growth equilibrium. See King et al. (1988) and Palivos et al. (1997).

<sup>&</sup>lt;sup>6</sup> Notice that a sufficient condition for this is in turn  $\sigma > 1$ , an assumption which is used often in the literature of endogenous growth (see, for example, Lucas, 1990) and is also supported by empirical evidence (see, for example, Weber, 1975, and Hall, 1988).

<sup>&</sup>lt;sup>7</sup> Corsetti and Roubini (1996) analyze the case where the government is allowed to borrow and lend, within a closed economy framework.

Hence:8

$$g_{i} = \tau_{i}^{ii} r_{i} \left( W_{i} - \sum_{j=1}^{I, j \neq i} k_{i}^{j} \right) + \tau_{i}^{ji} \sum_{j=1}^{I, j \neq i} (1 - \tau_{j}^{ji}) r_{j} k_{i}^{j} + \tau_{i}^{ij} \sum_{j=1}^{I, j \neq i} r_{i} k_{j}^{i}, \quad \forall i \in I$$

$$(9)$$

where  $g_i$  denotes government spending in country i. The right-hand side of (9) gives total revenue raised every period, which consists of revenue raised by taxing (i) capital income originating in country i and owned by residents of country i (first term), (ii) capital income originating in all countries  $j \neq i$  and owned by residents of country i (second term), and (iii) capital income originating in country i and owned by residents of all countries  $j \neq i$  (third term). Under a source (residence) tax,  $\tau_i^{ji} = 0$  ( $\tau_i^{ij} = 0$ ) and hence the second (third) term in (9) vanishes.

### 2.3. *Firms*

The representative firm in each country is assumed to operate with a technology of the general form

$$y_i = k_i f(g_{wi}), \tag{10}$$

where  $y_i$  denotes per capita output and  $g_{wi} \equiv g_i/W_i$ . Furthermore, it is assumed that  $f(0) \ge 0$ , f' > 0, and f'' < 0. Thus, the technology is strictly increasing and linear in k, strictly increasing and strictly concave in  $g_w$ , and k and  $g_w$  are Edgeworth complementary. Note that according to (10), for any given level of capital, a change in spending on the productive input relative to aggregate domestic wealth affects output. The reason for this is that, since aggregate domestic wealth in each country includes assets owned by domestic residents abroad, we adopt a broad definition of the productive input which includes domestic infrastructure as well as enforcement of property and rule of law to protect the smooth functioning of the capital markets. It is important to note that along the balanced growth path characterized below, the ratio  $g_w$  is constant so that an increase in the level of wealth is fully matched by an increase in the level of government spending. Therefore, the policy instrument is the constant ratio  $g_w$ . Finally, to ensure the existence of an optimal tax or, equivalently, of an optimal government spending to wealth ratio, we assume  $\lim_{g_w \to 0} f(g_w) = \infty$  and  $\lim_{g_w \to 1} f(g_w) = 0$ .

We would like to emphasize that we make use of this production function for analytical convenience. This is particularly convenient when we allow for the endogenous solution for the world interest rate in the case of large countries. For example, an alternative formulation would be to use simply the level of g instead of the ratio  $g_w$  in the production function. However, in this case we would have to introduce labor as an additional private input so that we can

<sup>&</sup>lt;sup>8</sup> Henceforth, we drop the time index, t, whenever doing so does not result in confusion.

obtain the constant returns to scale with respect to the private inputs which is necessary for the existence of competitive markets. Such a case would be analytically cumbersome and, as is well known from the work of Benhabib and Farmer (1994), there would be transitional dynamics and global indeterminacy. By adopting our formulation, we view k as broadly consisting of physical and human capital (see, e.g. Barro, 1990).

The aggregate capital stock of country i is

$$k_i = \sum_{j=1}^{I} k_j^i,$$

the total measure of the capital which is located in country i and owned by the residents of all countries  $j \in I$ , including i. Profit maximization by competitive firms then gives the marginal physical product of capital located in country i ( = real interest rate in country i) as

$$\partial y/\partial k_i = f(g_{wi}) = r_i(g_{wi}),$$
 (11)

where the function  $r_i(g_{wi})$  is strictly increasing and strictly concave, given the properties of the function  $f(\cdot)$ . Thus, output can be written as

$$y_i = r_i(g_{wi})k_i. (12)$$

## 3. OPTIMAL FISCAL POLICY

The benevolent policy maker in country i solves a "restricted Ramsey planner" problem (see, e.g. Lucas and Stokey, 1983; Corsetti and Roubini, 1996). More specifically, the policymaker i takes all prices and quantities of country  $j \in I$ ,  $j \neq i$ , as given and chooses consumption, investment allocation, government spending and tax rates in order to maximize the representative individual's lifetime utility, (2), subject to the private budget constraint, (3), the consumer's optimality conditions, (5)–(8), the government budget constraint, (9), and the firm's optimality condition, (11).

Given the production function specified above, it is well known that the consumer's problem does not display any transitional dynamics and hence the growth rate of consumption and wealth at any time *t* is given by

$$\eta_i \equiv \frac{\dot{c}_i}{c_i} = \frac{\dot{W}_i}{W_i} = \frac{(1 - \tau_i^{ii})r_i - \beta}{\sigma}.$$
 (13)

Furthermore, using (3) and (6), the initial consumption level can be written as

$$c_i(0) = \frac{(\sigma - 1)(1 - \tau_i^{ii})r_i + \beta}{\sigma} W_i(0) > 0.$$
 (14)

Next, using (13) and (14), one can compute the indirect lifetime utility function of the representative agent from

$$V_{i} = \frac{1}{\beta(1-\sigma)} + \frac{1}{1-\sigma} \left[ \frac{\beta}{\sigma} + \frac{\sigma-1}{\sigma} (1-\tau_{i}^{ii}) r_{i} \right]^{-\sigma} [W_{i}(0)]^{1-\sigma}.$$
 (15)

Equation (15) can also be written as

$$V_{i} = \frac{1}{\beta(1-\sigma)} + \frac{1}{1-\sigma} \left[\beta + (\sigma - 1)\eta_{i}\right]^{-\sigma} [W_{i}(0)]^{1-\sigma}.$$
 (16)

Such an expression will be useful for the ranking of different equilibria, since by simple differentiation, one finds

$$\frac{\partial V_i}{\partial \eta_i} = \sigma [\beta + (\sigma - 1)\eta_i]^{-\sigma - 1} [W_i(0)]^{1 - \sigma} > 0.$$
(17)

That is, a higher growth rate corresponds to a higher welfare level, and vice versa. By ranking different equilibria in terms of the growth rate (welfare) we can automatically rank them in terms of welfare (growth rate) as well.

Next, notice that the government's problem can now be reduced to choosing the tax rates,  $\tau_i^{ii}$ ,  $\tau_i^{ji}$  and  $\tau_i^{ij}$ , the government spending to wealth ratio,  $g_{wi}$ , and investment abroad,  $k_i^j$ ,  $\forall j \in I$  and  $j \neq i$ , in order to maximize (15), subject to the consumer's optimality conditions with regard to investment allocation, (6), the government budget constraint, (9), and the firm's optimality condition, (11); where it may be recalled that, to be consistent with the definition of a Nash equilibrium, we assume that in solving this optimization problem the policymaker of country i takes the quantities, such as  $k_j^i$ , the prices, such as  $r_i$ , and the tax rates in every other country j as given.

# 3.1. Autarky

First, we consider the case of an economy that has no interaction with the rest of the world, as a benchmark case; i.e.  $I=1, k_j^i=0, \forall i\neq j$ , and  $k_i^i=k_i=W_i$ . The restricted Ramsey problem faced by the policymaker is simple:<sup>9</sup>

$$\max_{g_w} V(g_w), \tag{P1}$$

where  $r = f(g_w)$  and  $\tau = g_w/r$ . It is straightforward to show (see Barro, 1990; Corsetti and Roubini, 1996) that in this case the optimal tax and spending policy is the Pigouvian solution given by

$$\frac{\partial r}{\partial g_w} = 1 \tag{18}$$

<sup>&</sup>lt;sup>9</sup> Notice that the indirect utility function, given in (15) or (16), depends *inter alia* on the initial government spending to wealth *ratio*,  $g_w(0)$ . Given that W(0) is predetermined, maximizing this indirect utility function with respect to  $g_w$  is equivalent to maximizing it with respect to g. Of course, this is true only for the specific endogenous growth case with no transitional dynamics examined in this model.

or

$$(1 - \tau) \frac{\partial r}{\partial \tau} = r. \tag{19}$$

Notice that, since  $\tau = g_w/r$ , after tax output is given by  $[1 - (g_w/r)]r(g_w)k$ . Differentiating this expression with respect to  $g_w$  results in (18), which requires that the marginal benefit of increasing government spending,  $\partial r/\partial g_w$ , be equal to its marginal cost, 1. Similar reasoning applies to (19). Furthermore, it follows from (17) that at the optimal ratio of spending to wealth or, equivalently, at the optimal tax rate

$$\frac{\partial \eta_i}{\partial g_{wi}} = 0.$$

Thus, the growth rate of consumption and wealth achieves its maximum as well.

## 3.2. Source-based tax

Next consider the case of a source-based tax system where  $\tau_i^{ji} = 0$ ,  $\forall i, j \in I$ ,  $j \neq i$ . Furthermore, we assume that capital income is taxed at the same rate, regardless of its ownership; that is;  $\tau_i^{ii} = \tau_i^{ij} = \mu_i$ ,  $\forall j \in I$  (see, e.g., Turnovsky and Bianconi, 1992; Bianconi, 1995). The government budget constraint, (9), can then be written as

$$g_{wi} \equiv \frac{g_i}{W_i} = \mu_i r_i \frac{k_i}{W_i}. \tag{20}$$

# A. Noncooperative equilibrium

The policymaker's problem can be formulated as follows:

$$\max_{\{\mu_i, k_i^j(0)\}} V_i \tag{P2}$$

subject to<sup>11</sup>

$$[(1 - \mu_i)r_i - (1 - \mu_j)r_i]k_i^j(0) = 0, \quad \forall j \in I \quad \text{and} \quad j \neq i$$

where, given (20), we can write  $r_i = f(\mu_i r_i(k_i(0)/W_i(0)))$ . It follows then easily that, in addition to the I-1 constraints, the first-order necessary conditions for a maximum include

<sup>&</sup>lt;sup>10</sup> See also Gordon (1986) for a discussion in a small open-economy context.

<sup>&</sup>lt;sup>11</sup> Note that we write the constraint as specified since (6) implies that either  $(1 - \mu_i)r_i - (1 - \mu_i)r_i = 0$  or  $k_i^j = 0$ .

<sup>&</sup>lt;sup>12</sup> Note that, for analytical convenience, we choose  $\mu$  as one of the instruments in (P2). Letting  $g_w$  instead be one of the instruments and then determining  $\mu$  through (20) gives the same results.

$$\left[-r_i + (1 - \mu_i)\frac{\partial r_i}{\partial \mu_i}\right] \left(\theta_i - \sum_{j=1}^{I,j \neq i} \zeta_i^j k_i^j(0)\right) = 0$$
 (21)

$$\theta_{\rm i}(1-\mu_i)\frac{\partial r_i}{\partial k_i^j(0)}$$

$$+\sum_{j=1}^{I,j\neq i} \zeta_{i}^{j} \left[ (1-\mu_{j}) \left( r_{j} + \frac{\partial r_{j}}{\partial k_{i}^{j}(0)} k_{i}^{j}(0) \right) - (1-\mu_{i}) \left( r_{i} + \frac{\partial r_{i}}{\partial k_{i}^{j}(0)} k_{i}^{j}(0) \right) \right] = 0,$$
(22)

where

$$\theta_i = \left[ \frac{\beta}{\sigma} + \frac{\sigma - 1}{\sigma} \left( 1 - \mu_i \right) r_i \right]^{-\sigma - 1} [W(0)]^{1 - \sigma}$$

and  $\zeta_i^j$  denotes the Lagrange multiplier associated with the *j*th constraint. Similar conditions characterize the optimization problems faced by the other I-1 policymakers.

Next, note that with source-based taxation, in equilibrium,  $(1 - \mu_i)r_i = (1 - \mu_j)r_j$ . Combining this with (21) and (22) yields the following equilibrium values:

$$\mu_i = \mu, \quad \forall i \in I, \quad \text{where } (1 - \mu) \frac{\partial r}{\partial \mu} = r,$$
 (23)

$$\frac{k_i(0)}{W_i(0)} = 1, \quad \forall i \in I. \tag{24}$$

Thus, with a source-based tax and without any cooperation, the equilibrium values of  $\mu_i$  and  $k_i$  coincide with those obtained in the case of an autarkic economy. Equation (24) requires an initial reshuffling of the capital holdings on the portfolio of the consumer. This initial reshuffling ensures that the new equilibrium is along a balanced growth path with no net capital flows, but with nontrivial gross capital flows. Furthermore, the aggregate capital stock located in country i at time t is given by  $k_i(t) = k_i(0)\exp(\eta t) = W_i(0)\exp(\eta t)$ . <sup>13</sup>

## B. Cooperative equilibrium

Next suppose that a world policymaker maximizes a weighted average of each country's representative individual lifetime utility with respect to  $\{\mu_i, k_i^j(0)\}_{i,i=1}^{I,i\neq j}$ . More specifically, consider the following optimization

 $<sup>^{13}</sup>$  Note that, for the interest rate to be constant along the balanced growth path, the growth rate of capital must equal that of wealth ( $\eta$ ).

problem:

$$\max \sum_{i=1}^{I} \phi_i V_i \tag{P3}$$

subject to

$$(1 - \mu_1)r_1 = (1 - \mu_i)r_i$$
,  $i = 2, 3, ... I$ ,

where  $\phi_i \in (0, 1)$  and  $\sum_{i=1}^{I} \phi_i = 1$ . One can easily show that the first-order conditions for (P3) are the same as the ones for (P2); i.e. the solution to the cooperative problem coincides with that of the noncooperative and is given by (23) and (24).

To sum, with source-based tax financing, the cooperative and noncooperative equilibrium coincide and each country fully internalizes the impact that its spending has on all other countries through the world interest rate. The reason is that with source-based tax, a policymaker, who tries to maintain an after-tax interest rate above that of all other countries, faces the same tradeoff as in a closed economy (notice that an increase in the tax rate,  $\mu$ , has a positive  $(=(1-\mu)(\partial r/\partial \mu))$  and a negative effect (=r) on the after-tax interest rate).<sup>14</sup>

### 3.3. Residence-based tax

Next we analyze the noncooperative and cooperative equilibrium in the case of a residence-based tax. Accordingly,  $\tau_i^{ij}=0, \ \forall i,\ j\in I,\ j\neq i.$  Furthermore, we assume that all residents' income is taxed at the same rate regardless of its source; that is, for tax purposes, there is no distinction between investment at home and investment abroad. Hence,  $\tau_i^{ii}=\tau_i^{ji}=\tau_i,\ \forall i,\ j\in I.$ 

The government budget constraint can then be written as

$$g_{wi} = g_i / W_i = \tau_i r_i. \tag{25}$$

## A. Noncooperative equilibrium

To analyze the noncooperative equilibrium in the case of a residence-based tax, it is convenient to express  $V_i$  in terms of  $g_{wi}$ . Thus, using equation (25), we formulate the problem faced by country i's policymaker as

$$\max_{g_{wi}} V_i = \frac{1}{\beta(1-\sigma)} + \frac{1}{1-\sigma} \left[ \frac{\beta}{\sigma} + \frac{\sigma-1}{\sigma} \left( r_i(g_{wi}) - g_{wi} \right) \right]^{-\sigma} [W_i(0)]^{1-\sigma}$$
(P4)

subject to

$$0 \ge -r_i(g_{wi}) + r_j(g_{wj}), \quad \forall j \in I, j \ne i,$$

<sup>&</sup>lt;sup>14</sup> Note that our results are in contrast to the results of Keen and Marchand (1997), who find inefficiencies in the provision of the public input under source-based taxation. Our results are different because in their model governments take the return to capital as exogenously given.

where  $r(g_w) = f(g_w)$  (see equation (11)).<sup>15</sup> The Lagrangian function for (P4) can be written as

$$L_i = V_i(g_{wi}) - \sum_{j \neq i, j=1}^{I} \gamma_i^j (-r_i(g_{wi}) + r_j(g_{wj})),$$

where  $\gamma_i^j$ ,  $j \in I$ ,  $j \neq i$  denotes the nonnegative Lagrange multiplier of country i associated with the interest rate of country j. Notice that, by choosing its share of spending to wealth, each government tries to maintain a domestic interest rate which is greater than or equal to that of all other countries. This provides the channel for strategic interaction among countries to occur in this model. In particular, since a decrease in  $g_{wj}$ ,  $j \neq i$ , relaxes the constraint in (P4), the welfare level of country i is nonincreasing in  $g_{wj}$ ,  $\forall j \in I$ ,  $j \neq i$ . Hence, in general, the game exhibits percent negative spillovers (see, e.g. Cooper and John, 1988).

The first-order necessary conditions for (P4) are

$$\frac{\partial r_i}{\partial g_{wi}} = \frac{\theta_i}{\theta_i + \sum_{i=1}^{I,j \neq i} \gamma_i^j},\tag{26}$$

and

$$0 \ge -r_i(g_{wi}) + r_i(g_{wi}), \quad \gamma_i^j \ge 0, \quad \forall j \in I, j \ne i, \tag{27}$$

with complementary slackness.

Conditions similar to (26) and (27) characterize the optimization problems faced by the other I-1 governments. Combining all these conditions, we have that the Nash equilibrium is described by

$$1 \ge \frac{\partial r_i}{\partial g_{wi}} = \frac{\theta_i}{\theta_i + \sum_{j=1}^{I, j \ne i} \gamma_i^j}, \quad \forall i = 1, ..., I$$
 (28)

$$r_1(g_{w1}) = r_i(g_{wi}), \quad \forall i = 2, ..., I$$
 (29)

$$\gamma_i^j \geqslant 0, \quad \forall i, j \in I, i \neq j.$$
 (30)

In general, these conditions form a system of 2I - 1 equations in 2I unknowns,  $g_{wi}$ ,  $\sum_{j=1}^{I,j \neq i} \gamma_i^j$ ,  $\forall i = 1, ... I$ . Hence, the equilibrium is in general indeterminate. More specifically, equations (29) imply  $g_{w1} = g_{wi}$ ,  $\forall i \geq 2$ . That is, given that all other governments spend a certain share of wealth, country i finds it to its best

<sup>&</sup>lt;sup>15</sup> Note that specifying the constraint as  $[r_i(g_{wi}) - r_j(g_{wj})]k_i^j(0) = 0$  and expanding the set of choice variables to  $\{g_{wi}, k_i^j(0) | j \in I, j \neq i\}$  leads to exactly the same results.

interest to spend an equal share, but the actual magnitude of that share is indeterminate. <sup>16</sup> In fact, the game accepts a continuum of equilibrium vectors  $\mathbf{g}_{\mathbf{w}} \in \Re^{I}$  the coordinates of which are all identical,  $g_{w}$ , and take values in the interval  $[g_{w}^{L}, g_{w}^{U}]$ , with  $g_{w}^{L}$  being the solution to the equation  $\partial r/\partial g_{w} = 1$  and  $g_{w}^{U}$  being the share of spending to wealth that corresponds to  $\tau = 1$  or, equivalently, the solution to the equation  $r(g_{w}) = g_{w}$  (a fixed point of the function  $r(\cdot)$ ). <sup>17</sup> Over this interval:

$$1 \geqslant \frac{\partial r_i}{\partial g_{wi}}.$$

Furthermore, these equilibria are Pareto-ranked; the welfare of each country decreases in  $g_w$ . To see this notice that

$$0 \ge \frac{\partial V_i}{\partial g_w} = \sigma \left[ \frac{\beta}{\sigma} + \frac{\sigma - 1}{\sigma} \left( r(g_w) - g_w \right) \right]^{-\sigma - 1} \left[ W_i(0) \right]^{1 - \sigma} \left[ \frac{\partial r}{\partial g_w} - 1 \right]$$

and

$$0 \geqslant \frac{\partial \eta_i}{\partial g_w}$$

over the interval  $[g_w^L, g_w^U]$ , with the equalities holding at  $g_w = g_w^L$  (the autarky outcome).

<sup>16</sup> Note, however, that the multipliers,  $\gamma_j^i$ , which measure the magnitude of the negative spillovers faced by each country, are not necessarily the same across countries. More specifically, consider two countries m and n, which have the same preferences and technology. Since  $g_{wm} = g_{wn}$ ,  $\partial r_m(\cdot)/\partial g_{wm} = \partial r_n(\cdot)/\partial g_{wn}$ . Substituting this in (28) leads to

$$\sum_{j=1}^{I,j\neq m} \gamma_m^j = \frac{\theta_m}{\theta_n} \sum_{j=1}^{I,j\neq n} \gamma_n^j,$$

where

$$\frac{\theta_m}{\theta_n} = \left[\frac{W_m(0)}{W_n(0)}\right]^{1-\sigma}.$$

Thus, if the two countries are identical in every aspect then  $\sum_{j=1}^{I,j\neq m} \gamma_m^j = \sum_{j=1}^{I,j\neq n} \gamma_n^j$ . If, on the other hand, the two countries start from different initial conditions then the multipliers are weighted by the initial wealth differences. In this case, even though the two countries devote the same share of wealth on government outlays, they achieve different levels of welfare.

<sup>17</sup> The fact that  $g_w^U > g_w^L$  follows from

$$1 = \frac{\partial r(g_w^L)}{\partial g_w} = \frac{r(g_w^U)}{g_w^U} > \frac{\partial r(g_w^U)}{\partial g_w},$$

where the inequality is obtained from the strict concavity of  $r(\cdot)$  and the fact that  $r(0) \ge 0$ .

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The following subsection shows that there is a *coordination failure* (see, e.g. Cooper and John, 1988). By coordinating their actions, governments can achieve the efficient outcome given by  $\partial r_i/\partial g_{wi} = 1$  or  $g_{wi} = g_w^L$ ,  $\forall i \in I$ .

## B. Cooperative equilibrium

The cooperative equilibrium is the solution to the following optimization problem:

$$\max \sum_{i=1}^{I} \phi_i V_i(g_{wi}) \tag{P5}$$

subject to

$$r_1(g_{w1}) = r_i(g_{wi}), \quad \forall i \ge 2$$

where, once again,  $r(g_{wi}) = f(g_{wi})$ ,  $\forall i \in I$ . In addition to the I-1 constraints, the first-order necessary conditions include

$$\phi_1 \theta_1 \left( \frac{\partial r_1}{\partial g_{w1}} - 1 \right) + \frac{\partial r_1}{\partial g_{wi}} \sum_{i=2}^{I} \gamma_i = 0$$
 (31)

$$\phi_i \theta_i \left( \frac{\partial r_i}{\partial g_{wi}} - 1 \right) + \gamma_i \frac{\partial r_i}{\partial g_{wi}} = 0, \quad i \ge 2$$
 (32)

where  $\gamma_i$  denotes the Lagrange multiplier associated with *i*th constraint. Combining the first-order conditions leads to the following system:

$$r_1(g_{w1}) = r_i(g_{wi}), \quad \forall i \ge 2$$

$$\sum_{i=1}^{I} \frac{1}{(\partial r_i / \partial g_{wi})} \phi_i \theta_i \left( \frac{\partial r_i}{\partial g_{wi}} - 1 \right) = 0.$$

This system of I equations in I unknowns accepts the following unique solution:

$$g_{wi} = g_w^L, \quad \forall i \in I. \tag{33}$$

Given then (33), it follows from (31) and (32) that  $\gamma_i = 0$ , and  $\partial r_i/\partial g_{wi} = 1$ ,  $\forall i \in I$ . That is, the effect of each country's government spending share abroad is fully internalized. Thus, under no cooperation countries are likely to spend and tax over and above the optimal level; i.e.

$$g_{wi}\mid_{NC} \geqslant g_{wi}\mid_{C} = g_{w}^{L}, \quad \tau_{i}\mid_{NC} \geqslant \tau_{i}\mid_{C} = \tau^{L},$$

where the inequality regarding the tax rates follows easily from the fact that  $r(g_{wi}) > 0$  if  $g_{wi} > 0$  and the strict concavity of  $r(\cdot)$ .<sup>18</sup>

<sup>&</sup>lt;sup>18</sup> These results are consistent with the results in Turnovsky (1988), which are derived in a different framework.

Finally, the welfare and growth comparisons of these equilibria are also easily obtained. For countries starting with the same or different initial condition, we have

$$\eta_i \mid_C \geqslant \eta_i \mid_{NC}, \quad V_i \mid_C \geqslant V_i \mid_{NC}, \quad \forall i \in I.$$

In summary, with residence-based tax financing, we find an overprovision of the public input in the case of noncooperative behavior among policymakers. The intuition is that, in an attempt to maintain a before-tax interest rate above that of all other countries, each government determines its share of wealth spent on infrastructure without taking into account the effects of that spending on the world interest rate and hence on all other countries. Furthermore, moving from noncooperation to cooperation implies that all countries reduce their government expenditure shares and tax rates, achieving the maximum growth rate and welfare level; i.e. all countries are better off.

It is only in the case of noncooperative behavior that results are inconsistent with the traditional view of residence-based taxation; see, for example, Hamada (1966) and Diamond and Mirrless (1971). According to this view, residence-based taxation implies capital—export neutrality and thus an efficient allocation of capital across countries. However, this is the case only when taxes are financing certain types of government spending that do not enter the private production function. (In terms of our notation, this is the case when  $f(g_w) = 1$  and  $\partial r/\partial g_w = 0$ ). When there are productive public inputs, residence taxation can give an efficient allocation only if there is full coordination among players (policymakers).

# C. Example

The following functional form is isomorphic to the one used in the literature, such as Barro (1990) and Devereux and Mansoorian (1992):  $f(g_{wi}) = A(g_{wi})^{\alpha}$ , where  $A \in (0, \infty)$  and  $\alpha \in (0, 1)$ . For this particular case, closed-form solutions are easily obtained. More specifically solving the equations  $\partial r/\partial g_w = 1$  and  $r(g_w) = g_w$  yields, respectively,  $g_w^L = (\alpha A)^{1/(1-\alpha)}$  and  $g_w^U = A^{1/(1-\alpha)}$ . The corresponding tax rates are  $\tau^L = \alpha$  (the solution to the equation  $(1-\tau)(\partial r/\partial \tau) = r$ ) and  $\tau^U = 1$ . The set of noncooperative equilibria, S, then is

$$S = \{ \mathbf{g}_{\mathbf{w}} \in \Re^{I} \mid \mathbf{g}_{\mathbf{w}} = g_{w} \sum_{i=1}^{I} \mathbf{e}^{i}, i \in I \quad \text{and} \quad g_{w} \in [(\alpha A)^{1/(1-\alpha)}, A^{1/(1-\alpha)}] \},$$

where the vector  $\mathbf{e}^i = (0, ..., 1, ... 0)$  consists entirely of zeros except for 1 in its *i*th position. The unique cooperative equilibrium, on the other hand, is the vector  $\mathbf{g}_{\mathbf{w}}^{\mathbf{L}} = (\alpha A)^{1/(1-\alpha)} \sum_{i=1}^{I} \mathbf{e}^i$ .

#### 3.4. Discussion

As noted in the introduction, an important paper in the literature is Razin and Sadka (1991), which shows that a residence-based taxation system preserves the Diamond and Mirrless production efficiency theorem whereas a source-

based system does not. This result is in sharp contrast with the one derived in the present paper. It is important therefore to understand how the special features contribute to the new result. More specifically, the model that was developed here differs from the one in Razin and Sadka in the following respects: (1) it does not consider any endogenous labor–leisure choice. (2) It does not consider any immobile factors. In particular one can view k as a composite of physical and human capital, as in Barro (1990). In that case, labor is an internationally mobile factor, contrary to Razin and Sadka (1991). (3) The world lasts forever instead of just two periods and all private agents and governments have effectively an infinite horizon. (4) In this paper, government spending enters the production function instead of the utility function. (5) Countries are not small and do not take the interest rate as given: i.e. the world interest rate is determined endogenously. (6) The production function is linear in the accumulated input k. This is necessary for the existence of perpetual endogenous growth. (7) All countries are identical except perhaps from their initial conditions.

Nevertheless, the most important differences among those listed are the last four. The fourth difference (public services in the production function) induces governments to face a tradeoff between the cost and benefit of providing public services, which is described by the net interest rate. The fifth difference (large country assumption) imposes additional (external) constraints (notice that if governments take the world interest rate as given, as in other papers in the literature, then there is no strategic interaction among governments). The sixth and seventh differences (linearity and symmetry) are crucial for the indeterminacy of the equilibrium.

More specifically, in the present paper, the government of country i tries to maximize the welfare of the representative agent subject to an external constraint. In order to maximize welfare it must balance the marginal benefit,  $(1-\tau_i)\partial r_i/\partial \tau_i$ , with the marginal cost  $(r_i)$  of public services. This internal tradeoff is captured by the  $(1 - \tau_i)r_i$  term in the expression that gives the growth rate. The external constraint  $(1 - \tau_i)r_i \ge (1 - \tau_i)r_i$ , on the other hand, describes the same tradeoff. That is why solving the problem with and without the constraint gives the same solution (in the case of a source-based tax the constraint is not binding). Hence, the case of a source-based tax yields the same solution as the autarky case. Put differently, in the case of a source-based tax there is no strategic interaction among governments. Nevertheless, this is not so in the case of a residence-based tax where the government tries to balance the marginal benefit,  $(1 - \tau_i)\partial r_i/\partial \tau_i$ , with the marginal cost  $(r_i)$ , while at the same tax maintain a before-tax interest rate above that of all other countries,  $r_i \ge r_i$ . Notice that maximizing  $(1 - \tau_i)r_i$  is not the same as maximizing  $r_i$ . Thus, in this case the external constraint is binding.

Given these results, an obvious question then arises. If the residence-based tax system is inferior to its source-based counterpart, which is in turn equivalent to autarky, then why should the economy open itself in the first

<sup>&</sup>lt;sup>19</sup> See the discussion in section 2.3 above.

place? Of course, in the real world there are additional benefits from trade that are not captured in this model. Furthermore, this paper crucially restricts the analysis to a special case, that of symmetric countries. As indicated above, it is because of the linear production function and symmetry of technologies that we obtain indeterminacy and the possibility that a noncooperative equilibrium under residence-based tax is inferior to autarky.<sup>20</sup> In particular, notice the following results obtained in this paper for the case of symmetric linear technology with public service input. In the case of a closed economy, the optimal tax/government spending undo the distortion of the public service input and we obtain the Pigouvian first best. Opening up the economy for capital flows requires a regime of capital income taxation. Under source-based taxation the external constraint is not binding, an optimal choice of tax/government spending in the noncooperative and cooperative cases induces an initial reshufling of capital stocks so that the first best is achieved, exactly as in the case of autarky in section 3.1. Under residence-based taxation the external constraint is binding and the government of each country must satisfy the same equation as the one under autarky plus maintaining before-tax interest parity, i.e. it tries to maximize the after-tax interest rate and maintain the before-tax interest parity simultaneously. Thus, with residence-based taxation, noncooperative tax/government spending yields overprovision of the public input. In this case, opening up for capital flows is welfare-deteriorating because each government tries to set the own before- and after-tax interest rates above the other countries, without recognizing that they have an effect abroad. Hence, it is indeed because of the symmetric linear production technology identical across countries and the public service input that our results obtain.

Finally, it should be noted that our result regarding the inefficiency of the residence-based tax system is in accord with two other recent papers, Razin and Yuen (1997) and Razin et al. (1998), in which the competitive equilibrium may also be inefficient. The first paper shows that in the presence of human capital externalities  $\hat{a}$  la Lucas (1988), the optimal tax levied by the host country on migrant workers (nonresidents) is positive, which implies that the residence principle is no longer efficient. In a similar vein, the second paper shows that when domestic lending institutions observe a productivity shock before they make their loan decisions, but foreign lending institutions do not (i.e. there exists asymmetric information), then it may be optimal to either tax or subsidize nonresident capital-investors.

## 4. CONCLUSIONS

We have analyzed a dynamic one-good, multicountry model with perfect capital mobility and productive government spending to study fiscal

<sup>&</sup>lt;sup>20</sup> Using a different framework, Helpman and Razin (1983) and Razin et al. (1999) also find that opening up the economy may lead to welfare losses.

<sup>&</sup>lt;sup>21</sup> We would like to thank a referee for bringing this point to our attention.

interdependence among countries. We have found that, in the case of a sourcebased tax, there is no strategic interaction and that the noncooperative equilibrium is efficient. In the case of a residence-based tax, however, there is strategic interaction and in the absence of cooperation countries tend to spend over and above the optimal Pigouvian level. Furthermore, the noncooperative equilibrium is not unique.

We think that this is an important area for future research. Nevertheless, for the sake of brevity, we mention only a few possible extensions. First, one can introduce uncertainty associated with investment abroad. This will equate the interest rates only in an expected-value sense and can potentially determine the level of foreign investment that each country undertakes. Second, one may want to introduce heterogeneity not only with respect to initial conditions but also with respect to preferences or technology, This will most likely yield a unique equilibrium where the share of government spending to wealth differs across countries. Finally, another extension can be based on a reinterpretation of government spending so that each government's expenditure has a direct impact domestically as well as abroad. This will introduce additional channels of strategic interaction and may also be proved useful in determining a unique noncooperative equilibrium under perfect capital mobility and residence-based tax schemes.

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