

HETEROGENEITY, EFFICIENCY AND ASSET ALLOCATION WITH ENDOGENOUS LABOR SUPPLY: THE STATIC CASE*

by

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We study the implications of consumption and labor allocations with *ex ante* efficiency and possibly *ex post* inefficiency on international/interregional portfolio diversification. The answers we obtain depend crucially on the market regime relative to unemployment insurance. If there are complete markets for unemployment insurance, changes in asset allocation are small in the presence of *ex post* inefficiency, but if there are incomplete markets for unemployment insurance, changes in asset allocation can be large. The direction of the asset movement is towards more diversification.

(his) natural honesty helped others to avoid falling into the same traps he had fallen into, by signposting the danger areas . . .

(Gribbin and Gribbin, 1998, p. 159)

1 INTRODUCTION

In this paper, we consider simple general equilibrium models with endogenous labor supply and potential for adverse selection based on private information of individual preferences. We use the models to draw conclusions about asset allocation by considering the equity holdings that replicate deviations in consumption from a nonstochastic equilibrium.

We study the implications of consumption and labor allocations with *ex ante* efficiency and possibly *ex post* inefficiency for international/interregional portfolio diversification. The results we obtain depend crucially on the market regime relative to unemployment insurance. If there are complete markets for unemployment insurance, the effects on asset allocation patterns are very small even in the presence of *ex post* inefficiency, but if there are incomplete markets for unemployment insurance then the effects on asset allocation patterns can be large.

The issue of portfolio choice with endogenous labor supply has been addressed by Bodie *et al.* (1992) in a partial equilibrium framework and by

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Jermann (1998) in a general equilibrium framework. However, none of these authors considers the case of private information and the potential adverse selection problem that arises as well as the possibility of incomplete insurance markets. On the other hand, Marcet *et al.* (1998) consider the issue of incomplete markets in unemployment insurance but do not consider the problem of asset allocation. This paper fills the gap in the literature by providing a first attack on the problem of asset allocation under general equilibrium with endogenous labor supply, private information and incomplete markets. In particular, we examine the specific asset allocation that replicates infinitesimally small random deviations from a nonstochastic equilibrium in the presence of private information in a multi-agent general equilibrium model with endogenous labor supply.

Realistically, a key motivation for the analysis is that labor is relatively immobile and unemployment insurance policies may vary significantly across regions or countries. Hence, the consequences on portfolio choice are worth studying. A more subtle motivation is the literature on involuntary unemployment based on pure market outcomes. Involuntary unemployment is consistent with *ex ante* market efficiency and asymmetric information, but there exist *ex post* gains to trade or *ex post* inefficiencies. In this context, what is an apparent market failure has a plausible market-based explanation (see for example Chari, 1983; Prescott and Townsend, 1984).¹ In this paper, we apply the same characterization to the problem of asset allocation. It is well documented that there is a lack of international portfolio diversification inconsistent with simple complete markets portfolio choice models (see for example French and Poterba, 1991; Leung, 1995; Lewis, 1996; Jermann, 1998). Our contribution is to examine whether *ex ante* market efficiency with asymmetric information can lead to *ex post* inefficiencies that can explain the apparent lack of international asset diversification when labor supply is endogenous and tradable. Our main positive result is that, with endogenous labor supply, changes in asset allocation depend crucially on the market structure for unemployment insurance. In the absence of complete unemployment insurance, changes in asset allocations are large, but when complete unemployment insurance markets exist changes in asset allocations are negligible. However, when changes in asset allocation are large, the direction of change is towards more diversification.

The paper is organized as follows. Section 2 considers the basic structure and the benchmark of full information with homogeneous types. Section 3 examines the alternative models with different information and

¹It is well known that *ex ante* efficiency may be consistent with *ex post* inefficiency. For example, one of the major applications is in the literature on contracts with renegotiations, e.g. Hart (1995).

market structures. Section 4 examines the asset allocation problem and shows the main results, while Section 5 concludes.

2 BASIC STRUCTURE

The model in this paper is a one-good model cast in a static framework with a finite number of units indexed by $j \in \mathfrak{J}: j = 1, 2, \dots, J$. Each unit may be referred to as an island, a region or a country inhabited by a large (countably infinite) number of identical individuals. Hence, *a priori* there may be no differences within units, but potential differences across units.

A typical unit has a production technology given by

$$y_j = z_j f(n_j) \quad (1)$$

where y_j is the output produced by unit j , n_j are the number of hours spent on the production of the good, f is a strictly increasing and strictly concave function identical for all j ($f' > 0$, $f'' < 0$) and z_j is the productivity level of the technology. The differences in productivity across units may be potentially unobservable; however, we assume the existence of an organized asset market that reveals the market-relevant information of the unit's productivity as in the recent contribution of Berliant and De (1998).

In what follows, we assume that $f(n_j)$ takes the specific form

$$f(n_j) = (n_j)^\alpha \quad (2)$$

for $\alpha \in (0, 1)$ with returns to the variable and fixed factors given by

$$w_j = \alpha y_j \quad (3a)$$

$$e_j = (1 - \alpha)y_j \quad (3b)$$

respectively. Capital markets are perfectly integrated across units, labor is assumed immobile but labor income is assumed to be tradable (see for example Leung, 1995; Jermann, 1998).

Decisions by representative individuals in each unit are taken according to Fig. 1. When there is private information in individual preferences, *ex ante* equilibrium and *ex post* outcomes may differ relative to the point where all contracts are fulfilled, private information is revealed and production technology is realized. All decisions regarding consumption and labor supply are *ex ante*, whereas potential portfolio allocations that replicate equilibrium can be taken possibly *ex post*.

A typical unit utility function is given by

$$w^j(c_j, n_j) \equiv u^j(c_j) + v^j(n_j) \quad (4)$$

where c_j is the level of consumption of a representative individual in unit j . The function u is assumed to be strictly increasing and strictly concave

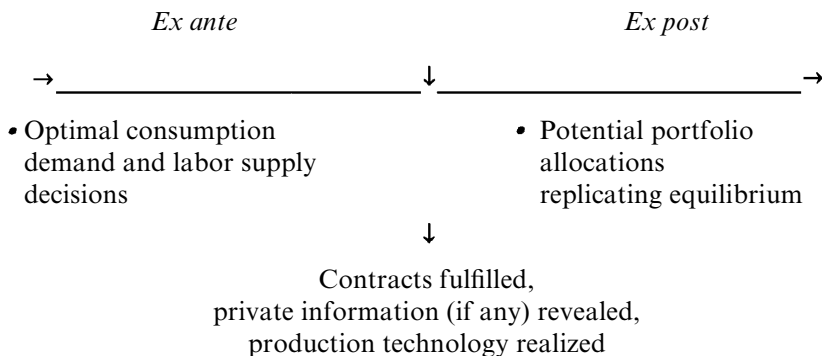


FIG. 1 Information and Decision Flows

($u'_1 > 0$, $u''_{11} < 0$), and the function v is assumed to be strictly decreasing and concave ($v'_1 < 0$, $v''_{11} \leq 0$). The main assumption in (4) is that consumption and labor are separable in utility. The source of private information in this model will be regarding the parameter $v''_{11} \leq 0$, i.e. the concavity of the utility function with respect to the labor supply.

The possible randomness in the model will be infinitesimally small. We shall consider infinitesimally small deviations from a nonstochastic equilibrium generated by z_j , i.e. dz_j , from $z_j = 1$. In particular, dz_j is generated randomly and enters the economic equilibrium linearly. This method, which has been used by Jermann (1998) and references therein, takes into account up to second-order derivatives of the functions above associated with first-order moments of the distributions (means), but ignores third-order derivatives associated with second-order moments (variances).

2.1 Full Information with Homogeneous Types

In the absence of heterogeneity across units there is no distinction between *ex ante* and *ex post* allocations, and $w^j = w$. A Pareto-efficient allocation can be obtained by maximizing the social welfare function subject to the resources constraints, or

$$\max_{\{c_j, n_j\}} E \left[\sum_j \omega_j w(c_j, n_j) \right] \tag{5}$$

subject to

$$\sum_j \pi_j [c_j - z_j f(n_j)] \leq 0$$

where ω_j are arbitrary welfare weights satisfying $\{\omega_j: \omega_j \geq 0, j = 1, 2, \dots, J, \sum_j \omega_j = 1\}$, π_j are resource weights to account for possible

differences in the size of units satisfying $\{\pi_j: \pi_j \geq 0, j = 1, 2, \dots, J, \sum_j \pi_j = 1\}$, and E is the expectations operator. In this framework, a solution to (4) yields Pareto efficiency *ex ante* and *ex post*, or

$$-v_1(n_j)/u_1(c_j) = z_j f'(n_j) \quad \text{for all } j = 1, 2, \dots, J \quad (6)$$

That is, the marginal rate of substitution between consumption and work in utility is equal to the marginal rate of transformation in production. Furthermore, if $\omega_j = \pi_j = 1/J$ for all j , then $c_j = c$ and $n_j = n$ for all j , the perfectly pooled equilibrium (e.g. Lucas, 1982; Leung, 1995).

3 HETEROGENEITY, EFFICIENCY AND MARKET COMPLETENESS

We proceed by examining alternative cases relating to the information structure about potential differences in preferences and market regimes.

3.1 Full Information with Heterogeneous Types

Consider heterogeneity in preferences across units, but full information, i.e. the heterogeneity is public information. Each unit has only one type and differences across units reflect differences across types. With perfect information, *ex ante* and *ex post* allocations are identical across units. Again, the Pareto-efficient allocation is obtained by maximizing the social welfare function subject to the resources constraints, or

$$\max_{\{c_j, n_j\}} E \left[\sum_j \omega_j w^j(c_j, n_j) \right] \quad (7)$$

subject to

$$\sum_j \pi_j [c_j - z_j f(n_j)] \leq 0$$

A solution to (7) yields Pareto efficiency *ex ante* and *ex post*, or

$$-v_1^j(n_j)/u_1^j(c_j) = z_j f'(n_j) \quad \text{for all } j = 1, 2, \dots, J \quad (8)$$

That is, the marginal rate of substitution between consumption and work in utility for unit type j is equal to the marginal rate of transformation in production for that type.

For example, let there be two units or types $J = 2$ with $\omega_j = \pi_j$ and preference structure

$$\begin{aligned} j = 1 & \quad v_{11}^1(n_1) = 0 \\ j = 2 & \quad v_{11}^2(n_2) < 0 \end{aligned} \quad (9)$$

Thus, for both types $j = 1, 2$, preferences are separable in consumption

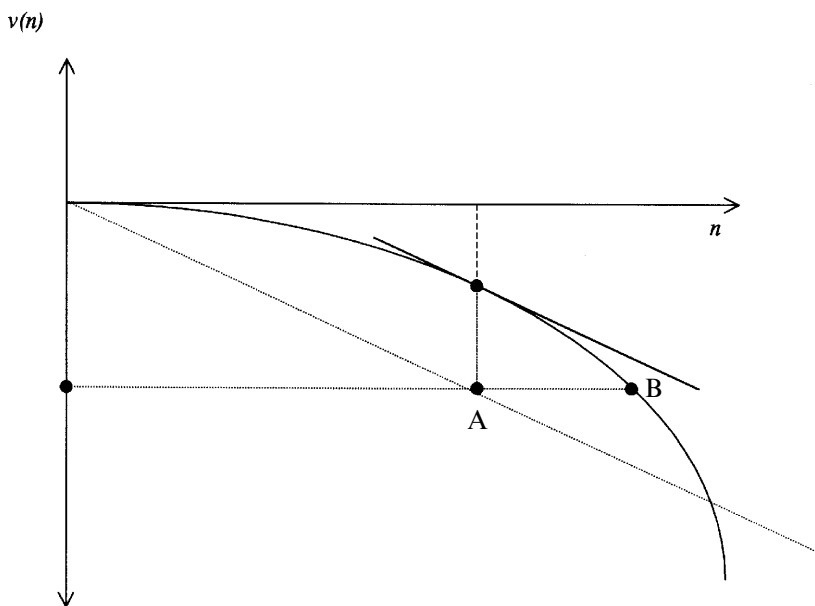


FIG. 2 Preferences for Labor

and labor supply, with identical forms in the consumption argument. For type $j = 1$ preference is linear in labor supply and for type $j = 2$ it is strictly concave in labor supply. This difference in preferences implies that for type $j = 1$ the elasticity of substitution between consumption and leisure is larger than for type $j = 2$. In particular, the linearity in labor supply implies that $j = 1$ is risk neutral in labor supply whereas the strict concavity for $j = 2$ implies risk aversion in labor supply. Figure 2 illustrates this difference: the more risk-averse individual, $v_{11}^2(n_2) < 0$ (the concave curve), will prefer to work more hours at point B than a lottery that would give expected disutility at point A, whereas the risk-neutral type, $v_{11}^1(n_1) = 0$ (straight line), is indifferent between the certainty and the gamble both at point A.

According to this preference structure, the optimal allocation across units takes the form

$$c_1 = c_2 \quad n_1 < n_2$$

Both types consume the same amount, but individuals in unit $j = 2$ work more hours since they are risk averse in labor supply whereas individuals in unit $j = 1$ are risk neutral and work less. It is optimal for type $j = 2$ to work more hours and shift risk to type $j = 1$.

3.2 Private Information in Individual Preferences with Heterogeneous Types and *ex post* Efficiency: Complete Markets

Now, introduce private information into the previous model. In this case, individual differences across units are private information of the specific unit and there may be differences in allocation *ex ante* versus *ex post*. A feasible and implementable allocation requires incentive compatibility constraints of the form

$$w^j(c_j, n_j) \geq w^j(c_i, n_i) \quad \text{for all } j, i, \in \mathcal{J}, i \neq j \quad (10)$$

That is, an individual of unit j when faced with alternative consumption–labor supply bundles will have an incentive to reveal his or her true type, i.e. an incentive not to misrepresent his or her preferences towards labor. It is clear that, with private information, the model in 3.1 above is such that the incentive compatibility constraints will be violated *ex ante* because both types consume the same amount and type $j = 2$ works more. Therefore, type $j = 2$ will have an incentive to misrepresent as type $j = 1$. This is a classic adverse selection problem. Technically, for u^j strictly concave in consumption, the consumption–labor supply possibility set is not convex (e.g. Prescott and Townsend, 1984).

The revelation mechanism used here to avoid the adverse selection problem is to introduce a lottery scheme that convexifies the consumption–labor supply possibility set.² Denoting the consumption–labor supply bundles $(c, n) \in \mathcal{L}$, where \mathcal{L} is the consumption–labor possibility set, with the introduction of the lottery scheme the incentive compatibility constraints analogous to (10) are

$$\sum_{(c,n) \in \mathcal{L}} \phi_j(c, n) w^j(c, n) \geq \sum_{(c,n) \in \mathcal{L}} \phi_i(c, n) w^j(c, n) \quad \text{for all } j, i \in \mathcal{J}, i \neq j \quad (11)$$

where $\phi_j(c, n) \geq 0$ and $\sum_j \phi_j(c, n) = 1$ is the lottery for bundle (c, n) . Hence, the incentive compatibility constraints in (11) are linear in the lottery and yield a convex consumption–labor supply possibility set. A consequence of introducing the revelation mechanism through the lottery scheme is that *ex ante* and *ex post* allocations may differ. *Ex ante* all individuals in all units are identical in expectations, but *ex post* the relevant differences are realized. For example, consider $J = 2$ with $\omega_j = \pi_j$, and preference structure as in (9). The revelation mechanism consists of introducing a lottery in the labor supply of individuals of unit $j = 1$ to make it unattractive to individuals of unit $j = 2$ who are risk averse, while not

²The scheme presented here is based on Prescott and Townsend (1984), and other applications of lotteries may be found in Rogerson (1988) and Besley *et al.* (1994). For a comprehensive exposition of general equilibrium with lotteries, see Townsend (1987).

affecting the decisions of $j = 1$ who are risk neutral. Let the lottery for $j = 1$ be a contract with the firm with the following terms:

$$\begin{array}{ll} \text{with probability } 1 - \phi & n_1 = 0 \\ \text{with probability } \phi & n_1 = \underline{n} > 0 \end{array}$$

for $\phi \in (0, 1)$, and $\underline{n} > 0$ given. The lottery ticket gives every holder full unemployment insurance; thus there are *complete markets* in unemployment insurance.³ The effective hours worked will be $\phi \times \underline{n}$ and every individual of unit $j = 1$ will receive *ex post* a full wage $z_1 f'(\phi \underline{n})$ whether working or not.

The expected utility for $j = 1$, *ex ante*, is $w^1(c_1, \phi \underline{n})$ and $j = 1$ maximizes expected utility by choice of probability ϕ . *Ex ante* allocations are obtained as solutions to the social problem

$$\max_{\{c_1, c_2, \phi, n_2\}} [\omega_1 w^1(c_1, \phi \underline{n}) + \omega_2 w^2(c_2, n_2)] \quad (12)$$

subject to

$$\pi_1 [c_1 - z_1 f(\phi \underline{n})] + \pi_2 [c_2 - z_2 f(n_2)] \leq 0$$

where in the resources constraint ϕ enters as the proportion of individuals of unit $j = 1$ who actually work. Note that the social planner knows the location of individuals across units, but due to private information must give individuals the right incentive to reveal truthfully. Hence, the contract is offered to all across units and gives the right incentive for all to reveal truthfully. In effect, there are no *a priori* intra-unit differences, and *ex ante* no inter-unit differences as well. However, *ex post* both intra-unit and inter-unit differences will arise. A solution to (12) yields Pareto efficiency *ex ante* for all units, or

$$-v_1^1(\phi \underline{n})/u_1^1(c_1) = z_1 f'(\phi \underline{n}) \quad (13a)$$

$$-v_1^2(n_2)/u_1^2(c_2) = z_2 f'(n_2) \quad (13b)$$

That is, *ex ante*, the marginal rate of substitution between consumption and work in utility for each unit or type is equal to the marginal rate of transformation in production for that unit or type. Indeed, this *ex ante* allocation is identical to the full information allocation with heterogeneity in 3.1 when we set

$$n_1 = \phi \underline{n} \quad (13c)$$

³We do not consider any potential moral hazard problem relating to the work effort in the presence of full insurance here. The papers by Hansen and Imrohoroglu (1992) and Atkeson and Lucas (1995) present models where the moral hazard problem in unemployment insurance is fully analyzed.

Hence, *ex ante* Pareto efficiency holds and the lottery makes everyone better off in expectations.

The *ex post* allocation in this case is also efficient. The individuals in unit $j = 1$ are subdivided into the fraction $1 - \phi$ who do not work, but due to the complete markets in unemployment insurance receive a full wage $z_1 f'(\phi \underline{n})$, and the fraction ϕ that work \underline{n} hours receiving the same wage $z_1 f'(\phi \underline{n})$. Thus, within and across units, individuals consume the same amount $c_1 = c_2 = c$. Because of the linearity of the utility function of all $j = 1$ in ϕ (risk neutrality), and separability between consumption and labor, we have that

$$v_1^1(\phi \underline{n}) = v_1^1(0) = \text{constant} \tag{14}$$

That is, it is independent of $\phi \underline{n}$ *ex post*. Thus, efficiency *ex post* holds for all $j = 1$, i.e.

$$-v_1^1(\phi \underline{n})/u_1^1(c_1) = -v_1^1(0)/u_1^1(c_1) = z_1 f'(\phi \underline{n}) \tag{15}$$

Efficiency holds as well for all $j = 2$, or

$$-v_1^2(n_2)/u_1^2(c_2) = z_2 f'(n_2) \tag{16}$$

To sum, for $v_{11}^1(n_1) = 0$, $v_{11}^2(n_2) < 0$, separable utility between consumption and labor, and private information, *ex ante* efficiency is consistent with *ex post* efficiency with lotteries as a revelation mechanism. There will be no differences in consumption within and across units, but there will be *ex post* differences in actual labor supply within unit $j = 1$ and across units as well.

3.3 Private Information in Individual Preferences with Heterogeneous Types and *ex post* Inefficiency: Complete Markets

Consider the model in 3.2 with $J = 2$ but with a slight modification in the preferences described in (9). Let $v_{11}^2(n_2) < v_{11}^1(n_1) < 0$. Therefore, individuals in unit $j = 1$ are uniformly less risk averse than $j = 2$ or similarly have a uniformly higher elasticity of substitution. The only difference from 3.2 above is that now both types are risk averse. Individual preferences are private information and the revelation mechanism is identical: introduce a lottery for $j = 1$ to make it unattractive for $j = 2$, the more risk averse, while acceptable to $j = 1$, the less risk averse. The important issue here is the difference across units and not the specific risk neutrality versus risk aversion *per se*.

For small risk aversion of $j = 1$, *ex ante* Pareto efficiency holds in this case as well: for all of $j = 1$ *ex ante* expression (13a) holds and for all of $j = 2$ *ex ante* expression (13b) holds. However, *ex post* allocations may not be the same.

The individuals in unit $j = 1$ are subdivided into the fraction $1 - \phi$

who do not work, but due to the complete markets in unemployment insurance receive a full wage $z_1 f'(\phi \underline{n})$, and the fraction ϕ that work \underline{n} hours and receive the same wage $z_1 f'(\phi \underline{n})$. Thus, within and across units, individuals consume the same amount $c_1 = c_2 = c$. But now, both are risk averse, implying that

$$v_1^1(\phi \underline{n}) \neq v_1^1(0) \quad (17)$$

ex post, since the marginal rate of substitution is a function of the labor supply when all are risk averse. For the proportion $1 - \phi$ of individuals in unit $j = 1$ who do not work, there will be *ex post* inefficiency, or

$$-v_1^1(0)/u_1^1(c_1) \neq z_1 f'(\phi \underline{n}) \quad (18)$$

For the proportion ϕ of individuals in unit $j = 1$ who do work, there is *ex post* efficiency, or

$$-v_1^1(\phi \underline{n})/u_1^1(c_1) = z_1 f'(\phi \underline{n}) \quad (19)$$

For all of $j = 2$, consumption is constant, $c_1 = c_2 = c$, and there is *ex post* efficiency as in (16).

Hence, in this case *ex ante* efficiency is consistent with *ex post* inefficiency at least for some in the population of $j = 1$.

3.4 The Incomplete Markets Case

In cases 3.2 and 3.3 above, we assumed that there are complete markets for unemployment insurance so that a lottery holder can receive a full wage in the case of unemployment. This presumes that markets provide full insurance at actuarially fair prices (e.g. Marcet *et al.*, 1998). In this section, we assume that there are no insurance mechanisms available for $j = 1$, i.e. there are *incomplete markets* in unemployment insurance. The individuals of unit $j = 1$ are faced with idiosyncratic uninsurable risk.

We apply the same revelation mechanism except that the lottery contract specifies that the individual who does not work *ex post* will *not* receive a payment. In this case, *ex ante* efficiency holds exactly as before, i.e. Pareto efficiency *ex ante* (in expectations) holds, but *ex post* allocations are inefficient and different even from case 3.3 above.

The fraction $1 - \phi$ of individuals in unit $j = 1$ who do not work will not be able to consume the same amount as the other fraction ϕ *ex post* since with incomplete markets they receive nothing in terms of wages. Therefore, the fraction $1 - \phi$ cannot consume (or consumes just a fixed endowment more realistically) and the fraction ϕ consumes *ex post* $c_{1,\phi} = c_1/\phi > c_1$, where c_1 is the allocation in 3.2 or 3.3.

Thus, in the case of incomplete markets for unemployment insurance there is *ex ante* efficiency as before but there is *ex post* inefficiency for some

of $j = 1$. The nature of the inefficiency includes the one discussed in equation (17) relating to the marginal disutility of labor, and in addition it includes inefficiency in the marginal utility of consumption, so that *ex post* $u_1^1(c_1) \neq u_1^1(0)$ since consumption for all individuals in unit $j = 1$ is not going to be identical *ex post*.

4 CONSEQUENCES FOR ASSET ALLOCATION

We consider infinitesimal risk as small deviations from a nonstochastic equilibrium induced by z_j , i.e. dz_j , from $z_j = 1$. First, we linearize the first-order necessary conditions for an interior equilibrium from a nonstochastic equilibrium for all the problems examined in 3.1–3.4, to obtain the small deviations dc_j and dn_j as a function of dz_j for all j .

Let $d\mathbf{c}$ be a column vector with dimension $1 \times J$ with elements $(dc_1, dc_2, \dots, dc_J)$ and similarly let $d\mathbf{n}$ and $d\mathbf{z}$ be column vectors with dimension $1 \times J$ with elements $(dn_1, dn_2, \dots, dn_J)$ and $(dz_1, dz_2, \dots, dz_J)$. The solution for the linearized system of first-order conditions takes the general form

$$d\mathbf{c} = \mathbb{C} d\mathbf{z} \tag{20a}$$

$$d\mathbf{n} = \mathbb{N} d\mathbf{z} \tag{20b}$$

where \mathbb{C} and \mathbb{N} are $J \times J$ gradient matrices evaluated at the nonstochastic equilibrium with $z_j = 1$. In general, the elements of \mathbb{C} and \mathbb{N} are a function of income and substitution effects through the parameters of preferences and technology. However, if income and substitution effects exactly cancel out, then \mathbb{N} is singular. Since in this paper we are interested in the contribution of endogenous labor supply to asset allocation, we rule out the case where income and substitution effects cancel out with \mathbb{N} nonsingular throughout.

Units cannot trade or observe the technology z_j directly. However, organized markets for equity trade are available where the equity is the profit of each unit given in (3b). Using (1) and (3b), small deviations dz_j induce small changes in equity values (at $z_j = 1$) given by

$$de_j = (1 - \alpha)[f(n_j) dz_j + f'(n_j) dn_j] \quad \text{for all } j \tag{21}$$

In general, letting $d\mathbf{e}$ be a column vector with dimension $1 \times J$ and elements $(de_1, de_2, \dots, de_J)$ we obtain

$$d\mathbf{e} = \mathbb{E}_1 d\mathbf{z} + \mathbb{E}_2 d\mathbf{n} \tag{22}$$

where \mathbb{E}_1 is a $J \times J$ diagonal matrix with elements $(1 - \alpha)f(n_j)$ and \mathbb{E}_2 is a $J \times J$ diagonal matrix with elements $(1 - \alpha)f'(n_j)$. Substituting above for $d\mathbf{n}$ from (20b) and solving for $d\mathbf{z}$ we obtain

$$d\mathbf{z} = [\mathbb{E}_1 + \mathbb{E}_2 \mathbb{N}]^{-1} d\mathbf{e} \tag{23}$$

for the case where the $J \times J$ matrix $\mathbb{E}_1 + \mathbb{E}_2\mathbb{N}$ is invertible. Equation (23) maps the sources of stochastic deviations into equity values available in the organized market for trade. Thus, even though technology is not observed, the equity market provides an observable variable (e.g. Berliant and De, 1998). Then, consumption deviation $d\mathbf{c}$ in (20a) can be mapped into the equity funds as

$$d\mathbf{c} = \{\mathbb{C}[\mathbb{E}_1 + \mathbb{E}_2\mathbb{N}]^{-1}\} d\mathbf{e} \quad (24)$$

and a deviation $d\mathbf{c}$ can be optimally supported by holding $\mathbb{C}[\mathbb{E}_1 + \mathbb{E}_2\mathbb{N}]^{-1}$ ‘shares’ of equity fund \mathbf{e} . Denoting elements of the asset allocation matrix $\mathbb{C}[\mathbb{E}_1 + \mathbb{E}_2\mathbb{N}]^{-1}$ by c_{ij} for all $i, j = 1, 2, \dots, J$, the net ‘foreign’ asset position of each unit (region or country) is given by

$$F_i = \sum_{j, i \neq j} c_{ij} - \sum_{j, i \neq j} c_{ji} \quad (25)$$

which denotes the shares held by type i on equity funds issued by units j minus the shares held by type j on equity funds issued by units i satisfying $\sum_i F_i = 0$. Recognizing that the level of the current account of each unit is just y_j minus c_j , we have that

$$CA_j = z_j f(n_j) - c_j \quad \text{for all } j \quad (26)$$

and using (20)–(24) we obtain deviations of the current account from the nonstochastic equilibrium as a function of the equity funds:

$$dCA = \{[1/(1 - \alpha)]\mathbb{I} - \mathbb{C}[\mathbb{E}_1 + \mathbb{E}_2\mathbb{N}]^{-1}\} d\mathbf{e} \quad (27)$$

where dCA is a column vector with dimension $1 \times J$ and elements ($dCA_1, dCA_2, \dots, dCA_J$), $1/(1 - \alpha)$ is a scalar and \mathbb{I} is a $J \times J$ identity matrix. The intuition for (27) is that $[1/(1 - \alpha)] \times \mathbb{I} \times d\mathbf{e}$ is the portfolio of tradable equities for each individual of unit j as can be seen from (3b), i.e. output in terms of equities. This can be subtracted from consumption in (24) yielding the current account directly. We denote the elements of the $J \times J$ matrix $[1/(1 - \alpha)]\mathbb{I} - \mathbb{C}[\mathbb{E}_1 + \mathbb{E}_2\mathbb{N}]^{-1}$ by ca_{ij} for all $i, j = 1, 2, \dots, J$ and each denotes the change in asset allocation that replicates the change in the unit’s current account, i.e. the matrix is a measure of gross capital flows across units.

Equations (24)–(27) are the main relationships that represent the asset allocation across units. For each case examined in Section 3 regarding the nature of private information and market completeness, that set of equations can be evaluated and comparisons across the different regimes governing private information and market completeness can be drawn.

We pursue a quantitative approach in drawing the comparisons.⁴ A

⁴Analytical comparisons are infeasible in this framework; see for example Leung (1995) for a case without private information where analytical results can be drawn.

nonstochastic equilibrium is computed with $J = 2$, $\omega_j = \pi_j = \frac{1}{2}$ and $z_j = 1$ for $j = 1, 2$. Preferences take the form

$$w^1(c_1, n_1) = \ln c_1 - \delta(n_1^{1+\gamma_1})/(1 + \gamma_1) \tag{28a}$$

$$w^2(c_2, n_2) = \ln c_2 - \delta(n_2^{1+\gamma_2})/(1 + \gamma_2) \tag{28b}$$

where $\delta > 0$, and $0 \leq \gamma_1 < \gamma_2$ gives the curvature of the utility function relative to labor supply. The technology is given by (1)–(2). The preferences in (28a) and (28b) allow us to examine all cases in Section 3 by varying essentially the parameter γ_1 given plausible choices of α, δ, γ_2 and \underline{n} . We use the plausible choices $\alpha = 0.6, \delta = 1.5, \gamma_2 = 0.75$ and $\underline{n} = 0.6$.

First, note that under full information and homogeneous types $c_1 = c_2 = c, n_1 = n_2 = n, c_{ij} = c, F_i = CA_j = 0$ for all $i, j = 1, 2, ca_{ij} = ca_{ji}$ for $i = j$ and $ca_{ij} = -ca_{ji}$ for $i \neq j$: a perfectly symmetric equilibrium as in 2.1 above (e.g. Lucas, 1982; Leung, 1995). Naturally, introducing an asymmetry in preferences introduces asymmetry in asset allocation and nontrivial net asset position and current account allocations.

Table 1 presents results for the alternative regimes. The values in each

TABLE 1
SIMULATED ALTERNATIVE REGIMES

	c_{ij}		F_i		CA_j	ca_{ij}	
Full information, heterogeneity \equiv private information, heterogeneity $\gamma_1 = 0$ $\gamma_2 = 0.75$	1.249	1.250	0.27	E-3	-0.144	1.250	-1.250
	1.249	1.250	-0.27	E-3	0.144	-1.249	1.249
Private information, heterogeneity, <i>ex post</i> inefficiency, complete markets $\gamma_1 = 0.25$ $\gamma_2 = 0.75$	1.250	1.250	-0.20	E-5	-0.073	1.249	-1.250
	1.250	1.250	0.20	E-5	0.073	-1.250	1.249
Private information, heterogeneity, <i>ex post</i> inefficiency, incomplete markets $\gamma_1 = 0.25$ $\gamma_2 = 0.75$	0.014	3.076	1.826		-0.073	2.486	-3.076
	1.250	1.250	-1.826		0.073	-1.250	1.249
Private information, heterogeneity, <i>ex post</i> inefficiency, incomplete markets $\gamma_1 = 0.5$ $\gamma_2 = 0.75$	0.392	2.369	1.113		-0.030	2.108	-2.369
	1.249	1.250	-1.113		0.030	-1.249	1.250

Note: Values are $c_{11}, c_{12}, c_{21}, c_{22}$ for c_{ij}, F_1, F_2 for F_i, CA_1, CA_2 for CA_j , and $ca_{11}, ca_{12}, ca_{21}, ca_{22}$ for ca_{ij} .

box represent the matrices defined above. The main result is that inefficiency *ex post* with complete markets does not give rise to any substantive change in asset allocation, whereas the presence of incomplete markets does, i.e. uninsurable idiosyncratic risk matters.

The first row of the table presents the cases in 3.1 and 3.2. The two allocations are the same with the implied value for the lottery given by $\phi = 0.517$, as shown theoretically in (13c) above. The asymmetry in preferences induces a very small bias towards holding assets of the risk-averse unit, $c_{11} = c_{21} = 1.249$, whereas the risk-neutral unit runs a current account deficit (in levels) with a positive net asset position. Diversification across the units is almost even. The change in the asset allocation that replicates the change in the unit's current account, ca_{ij} , shows the extent of the gross flows across the units when there are technology disturbances. To replicate the change in the current account, both individuals must go long on their own fund and short on the other fund so that the change in their own technology can be fully reflected in the current account.

The second row of the table presents case 3.3 *ex post*. There is very little qualitative and quantitative change relative to 3.1–3.2 in terms of the asset allocation. The current account *level* is about a half less than the previous case but the change in the net asset position is trivial. Thus, we conclude that *ex post* inefficiency with complete markets for unemployment insurance seems to have very little marginal impact on the asset allocation of individuals.

The third and fourth rows present case 3.4 *ex post* where there are incomplete markets for unemployment insurance. The third row is a direct comparison with the complete markets case in the second row. We obtain a large shift in holdings of the less risk-averse unit towards the fund of the more risk-averse unit. The 'share' holdings are $c_{11} = 0.014$, $c_{12} = 3.076$, so that the portfolio of $j = 1$ (who works and consumes *ex post*) consists of 0.5% in the own equity fund and 99.5% in the other unit equity fund. As a result, the net asset position of the less risk-averse unit increases sharply as well as the asset allocation that replicates the change in the current account. The actual level of the current account deficit does not change, but the marginal effects are rather large. The less risk-averse individual ($j = 1$) holds most of his or her portfolio in the fund of the other unit. This strong diversification result is due to the willingness to trade away the risk of own technology shocks that will have a large effect on the consumption of the individual who actually works; we recall that *ex post* $c_{1,\phi} = c_1/\phi > c_1$ in 3.4.

The fourth row shows an allocation when γ_1 increases from 0.25 to 0.5 with incomplete markets making the two units more alike in terms of risk aversion. The asset allocation moves in the expected direction. The exodus in holdings of the less risk-averse own asset to the other asset is relatively moderate.

The results show quite sharply that *ex post* inefficiencies consistent with *ex ante* Pareto-optimal allocations alone do not give rise to substantive changes in asset allocation when there are complete markets for unemployment insurance, i.e. when there is full insurance for idiosyncratic risk. The differences realized *ex post* are too small in terms of asset holdings because it is in the interest of *all* individuals of the less risk-averse unit to hold portfolios that guarantee the variations in the lottery probability, regardless whether they work or not *ex post*. When there are incomplete markets the story is quite different. *Ex post*, the individuals of less risk-averse type who do not work will not receive any payment and will be shut off from the capital market. Therefore, the less risk-averse individual who works has an incentive to hold an overwhelming fraction of his or her portfolio on the foreign equity fund, thus diversifying away the exposure to the lottery probability. Note that this result is obtained with tradable labor income, but even if income were nontradable it would still hold; see for example Leung (1995) and Jermann (1998) for the case of nontradable labor income with homogeneous types.

5 CONCLUSIONS

We examined the issue of private information, efficiency and market structure in a multi-unit general equilibrium model with endogenous labor supply and its relationship to asset allocation. The main result is that *ex post* inefficiency consistent with *ex ante* efficiency does not matter for asset allocation when idiosyncratic risks are insurable. But incomplete markets for unemployment insurance have sharp implications for asset allocation. Uninsurable idiosyncratic risks matter for asset allocation. The major effect is to induce diversification away from the own equity into other units' equity in order to diversify away the exposure to the employment lottery. Therefore, even though *ex post* inefficiencies can be consistent with *ex ante* efficiency in consumption and labor allocations with private information, the changes in asset allocation in the case of incomplete markets for unemployment insurance go in the direction of more diversification and thus do not explain the lack of international portfolio diversification observed in the real world.

Further research in the direction of dynamics and capital accumulation or the explicit introduction of moral hazard considerations seem all worth pursuing.

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