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Intertemporal Budget Policies and Macroeconomic Adjustment in Indebted Open Economies

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Abstract

This paper analyzes the role of government intertemporal budget policies in a growing open economy including nominal assets in the presence of an upward sloping supply of debt. This introduces transitional dynamics that influence the effects of government policy instruments on economic growth and the long term fiscal liability. It is shown that capital income taxes or a combination of tax-cum-expenditure or government expenditure alone can balance the long term intertemporal government budget constraint. However, those results are shown to depend critically upon the extent of distortion in capital flows brought about the upward sloping supply of debt.

1. Introduction

This paper evaluates fiscal and monetary policies in terms of the public's intertemporal tax liability, measured by the present value of future lump-sum taxes scaled by the domestic capital stock. A key question is the extent to which fiscal and monetary policy can be used to balance the intertemporal government budget constraint. Several authors have provided important answers in the closed economy context.[1] However, in an open economy context there are fewer contributions. Motivated by the recent difficulties of sovereign nations under financial distress to roll over their national debt obligations, we consider intertemporal government budget policies in a small open economy in the presence of an upward sloping supply of debt. This introduces a premium on the interest service paid to domestic and foreign creditors and makes the interest rate in a small open economy vary with the level of foreign indebtedness. As a higher (lower) foreign debt comes at a higher (lower) foreign interest rate, changes in government expenditure and capital income taxes that affect the level of foreign debt, also affects the equilibrium interest rates and growth rate of the economy. Hence, even though the private and public assets are denominated in terms of the foreign price level, the interest "premium" introduces transitional dynamics and convergence towards the balanced growth path where the growth rates of domestic capital and consumption are equated through an adjustment of the country's net foreign asset position.[2]

While the focus of our paper is on the desirability and plausibility of monetary and fiscal policies for budgetary purposes in the presence of an upward supply of debt, the framework may be associated with several other issues. Several authors have focused

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on Laffer-style effects of fiscal policy with particular attention to dynamic scoring.[3] In our small open economy with upward sloped supply of debt, the net foreign asset position adjusts endogenously to take the economy to its balanced growth path. This adds a new channel for the possibility of dynamic scoring and we show that it depends critically on the elasticity of supply of debt. Our open economy model provides insight into the twin deficit phenomenon by providing a direct link between the "twin" deficits within a framework of intertemporal solvency and endogenous growth, an important extension of the closed economy models. In this context, the main result is that the economy's long-run tax liability depends not only on the primary deficit net of inflation tax revenues, but also on the long-run accumulation of national debt in terms of the capital stock, as well as on the speed of adjustment to the long-run balanced growth path. The latter is a consequence of the fact that the economy borrows (and lends) subject to an upward-sloping interest rate relationship. Our novel result is that the effect of a cut in capital income tax depends on its spillover on the economy national borrowing. A decrease in the capital tax increases the growth rate, which increases foreign debt; this will decrease the long run liability because higher growth increases capital tax revenues and reduces the long term tax liability. Hence, in our framework, foreign deficits are negatively related to the long term liability of the government. This effect is enlarged in the case where a tax-cum-expenditure policy is used.

Finally, the model relates to a strand of the literature on the interest burden in indebted economies and the possibility of erosion of government debts. This is the focus of the work of Engen and Hubbard (2004), Aizenman and Marion (2011) and Hall and Sargent (2011). In our model, assets are denominated in foreign currency and domestic price level changes do not affect the value of the debt; however it does have an effect on future tax liabilities through the traditional inflation tax channel.[4]

2. The Model and Growth Equilibrium

In this section we outline the small open economy structure, which is based on the endogenous growth model of Bianconi and Fisher (2005) with the assumption of an upward sloping supply of debt. There is one-good and purchasing power parity (PPP) holds at all times. Let p represent the rate of domestic inflation, P is the domestic price level, e is the rate of depreciation of the domestic currency and starred variables are foreign counterparts. PPP corresponds to $p = p^* + e$. We impose nominal interest rate parity, but incorporate a "premium" term that is an increasing, convex function of the stock of the real national debt $z \equiv Z/P^*$ scaled by the domestic capital stock k, e.g. Van der Ploeg (1996), Turnovsky (1997):

$$i = i^* + e + v(z/k), \quad v' > 0, \quad v'' > 0, \tag{1a}$$

where z is the nominal stock of national debt, i represents the domestic national interest rate, while i^* is the exogenous world nominal interest rate. The real interest rate parity condition is

$$i - p = i^* - p^* + v\left(\frac{z}{k}\right) \equiv r\left(\frac{z}{k}\right), \quad r' > 0, \quad r'' > 0. \tag{1b}$$

A representative agent has preferences for consumption c and real money balances $m \equiv M/P$, where M is the nominal stock money balances of the separable logarithmic form: $U(c,m) = \log c + \gamma \log m$, $\gamma > 0$. The agent accumulates real international financial assets (debt) $b = B/P^*$, where the nominal stock of bonds B is deflated by the foreign price level. The real return on international assets corresponds to (1b), respectively, while the (negative) real return on domestic money equals $-(p^* + e)$. As a producer, the agent has access to a technology that is linear homogenous in the domestic capital stock, Ak, which, under appropriate conditions detailed below, can sustain on-going growth. We assume that real investment I incurs installation costs modeled according to the standard quadratic specification:[5]

$$\Phi(i,k) = I\left(1+\frac{hI}{2k}\right), \quad h>0 \tag{1c}$$

The representative agent's problem is formulated as to maximize

$$\int_0^\infty (\log c + \gamma \log m)e^{-\delta t}dt, \quad \delta > 0,$$

subject to:

$$\dot{m} + \dot{b} + I\left(1+\frac{hI}{2k}\right) = (1-k)\alpha k + r\left(\frac{z}{k}\right)b - c - (p^*+e) - T, \quad \text{and} \quad \dot{k} = I \tag{2}$$

where τ = capital (output) tax rate ($\tau \in [0,1]$), T = lump-sum taxes, δ = exogenous domestic rate of time preference and subject to initial conditions on the stocks of domestic capital, nominal domestic money, and real international bonds: $k(0) \equiv k_0 > 0$, $M(0) \equiv M_0 > 0$, $b(0) \equiv B_0/P_0^* > 0$. In performing the optimization, the agent also takes the real interest rate $r(z/k)$ as given. Standard techniques yield the following optimality conditions:

$$\frac{1}{c} = \lambda, \tag{3a}$$

$$1 + h\frac{I}{k} = q \Rightarrow \frac{1}{k} = \frac{\dot{k}}{k} = \frac{q-1}{h} \equiv \phi \Rightarrow k(t) = e^{\int_0^t \phi(s)ds}, \tag{3b}$$

$$\delta - \frac{\dot{\lambda}}{\lambda} = \frac{\gamma}{\gamma m} - (p^*+e) = r\left(\frac{z}{k}\right), \tag{3c}$$

$$\frac{(1-\tau)\alpha}{q} + \frac{(q-1)^2}{2hq} + \frac{\dot{q}}{q} = r\left(\frac{z}{k}\right), \tag{3d}$$

$$\lim_{t\to\infty} \lambda b e^{-\delta t} = \lim_{t\to\infty} \lambda m e^{-\delta t} = \lim_{t\to\infty} q\lambda k e^{-\delta t} = 0, \tag{3e}$$

where λ is the shadow value of international assets, $q \equiv q'/\lambda$ is the shadow value of domestic capital in terms of international assets, and ϕ denotes the economy's balanced growth rate to be determined below. Equations (3a–e) have the standard interpretations. We next turn to the domestic public sector and describe the relationships defining the evolution of its financial liabilities. The public sector sells debt to foreign and domestic investors, assumed to be perfect substitutes for private assets traded internationally. Consequently, it bears a real rate of return equal to $r(z/k) = i^* - p^* + v(z/k)$. In contrast, money balances issued by the public sector are held only by domestic residents and erode in value at the rate equal to $p^* + e$. The flow of the government budget identity then corresponds to:

$$\dot{a} + \dot{m} = G + r\left(\frac{z}{k}\right)a - T - \tau\alpha k - (p^* + e)m, \tag{4a}$$

where $a \equiv A/P^*$ is the real stock of government bonds evaluated in terms the exogenous foreign price level and G is real government expenditure. The evolution of government bonds is also subject to an initial condition corresponding to $a(0) = a_0 = A/P^* > 0$, where A denotes the nominal stock of government bonds in terms of foreign currency. To guarantee the intertemporal solvency of the public sector, we impose the following limiting condition on the path of government debt: $\lim_{t\to\infty}\lambda a e^{-\delta t} = 0$. We assume that government expenditure and lump-sum taxes are set proportional to output. For government expenditure, this implies that $G(t) = \bar{g}\alpha k(t)$, where $\bar{g}$ is the fraction of output devoted to public expenditures; while for lump-sum taxes, the fraction $\bar{T}(t)$ corresponds to $\bar{T}(t) = T(t)/\alpha k(t)$. Finally, we specify that the public sector follows a simple constant nominal money growth rule, i.e. it sets $\sigma = \dot{M}/M$, which implies that the evolution of the real money supply equals:

$$\dot{m} = (\sigma - p)m = (\sigma - p^* - e)m. \tag{4b}$$

Using the definition of real national debt, $z \equiv a - b$, combining (2b) and (4b) yields the expression for the current account balance, where we substitute $G(t) = \bar{g}\alpha k(t)$. Further substituting for $I = h^{-1}(q - 1)k$, we can express the current account balance in terms of q:

$$\dot{z} = (1 - \bar{g})\alpha k + c + \frac{(q^2 - 1)}{2h} + r\left(\frac{z}{k}\right)z. \tag{5}$$

We next develop the open economy growth equilibrium. For consumption and national debt, define: $\chi \equiv c/k$ and $\psi \equiv z/k$. The rates of growth of these ratios equal $\frac{\dot{\chi}}{\chi} \equiv \frac{\dot{c}}{c} - \frac{\dot{k}}{k}$, $\frac{\dot{\psi}}{\psi} \equiv \frac{\dot{z}}{z} - \frac{\dot{k}}{k}$. Calculating the time derivative of (3a) and combining with (3a) and (3c), we obtain:

$$\frac{\dot{C}}{C} = -\frac{\dot{\lambda}}{\lambda} = r\left(\frac{z}{k}\right) - \delta. \tag{6}$$

Next, we solve for the differential equation $\dot{\chi}$ for consumption–capital ratio:

$$\dot{\chi} = \left[r(\psi) - \delta - \frac{q-1}{h}\right]\chi. \tag{7a}$$

Using (3b) and (5) the differential equation for $\dot{\psi}$ is

$$\dot{\psi} = \chi + \frac{(q^2 - 1)}{2h} - \frac{(q - 1)}{h} - (1 - \bar{g})\alpha + r(\psi)\psi \tag{7b}$$

and the differential equation for Tobin's q is found directly as

$$\dot{q} = r(\psi)q - (1-\tau)\alpha - \frac{(q-1)^2}{2h}. \tag{8}$$

Expressions (7a,b) and (8) state the system describing the dynamics of the small open economy and employing standard methods, the saddlepath solutions for the consumption-capital, national debt-capital, and Tobin's q correspond to:

$$\chi - \tilde{\chi} = \frac{\tilde{\chi}v'(\tilde{\psi})}{\xi_1}\left[1 + \frac{\tilde{q}/h}{\left[r(\tilde{\psi}) - \frac{\tilde{q}-1}{h}\right] - \xi_1}\right]\cdot(\psi - \tilde{\psi}), \tag{9a}$$

$$q - \tilde{q} = -\frac{v'(\tilde{\psi})\tilde{q}}{\left[r(\tilde{\psi}) - \frac{\tilde{q}-1}{h}\right] - \xi_1}\cdot(\psi - \tilde{\psi}), \tag{9b}$$

where $\psi = \tilde{\psi} - (\tilde{\psi} - \psi_0)e^{\xi_1 t}$ is the stable solution for the national debt-capital ratio, and $\xi_1 < 0$ is the stable root of the system. Both arms are negatively sloped since a greater level of indebtedness lowers domestic wealth, which reduces consumption; and, a higher level of indebtedness relative to the domestic stock of capital raises the domestic real interest rate which, under arbitrage, requires a lower Tobin's q. The dynamics of domestic real money balances scaled by the domestic capital stock follow as

$$\dot{\mu} = \left[\sigma + r(\psi) - \frac{q-1}{h}\right]\mu - \gamma\cdot\chi. \tag{10a}$$

Linearizing (10a) about the steady-state equilibrium, substituting for the solutions (9a,b), and integrating the resulting expression subject to the tranversality condition yield the saddlepath solution for μ:

$$\mu = \frac{\gamma\cdot\chi}{\sigma+\delta} + \frac{\Omega\cdot(\tilde{\psi} - \psi_0)e^{\xi_1 t}}{\sigma+\delta-\xi_1}, \tag{10b}$$

where $\tilde{\mu} = \frac{\gamma\cdot\tilde{\chi}}{\sigma+\delta}$ is the steady-state value of real money-domestic ratio and $\Omega \equiv v'(\tilde{\psi})\left(\tilde{\mu} - \frac{\gamma\tilde{\chi}}{\xi_1}\right)\left[1 + \frac{\tilde{q}}{(\delta-\xi_1)}\right] > 0$. We observe that the latter implies that the long-run ratio of real money to consumption, $\left(\frac{\widetilde{m/k}}{\widetilde{c/k}}\right)$, is $\gamma/(\sigma+\delta)$. Finally, given our logarithmic preferences parameterization, using equations (10a,b) and integrating, we obtain a measure of discounted welfare scaled by the capital stock given by:

$$W = \frac{1}{\delta}(\log\tilde{\chi} + \gamma\log\tilde{\mu}) + \left(\frac{1}{\delta-\xi_1}\right)(w_1 + \gamma w_2)(\tilde{\psi} - \psi_0), \tag{11}$$

where $w_1 \equiv \frac{-v'(\tilde{\psi})}{\xi_1}\left[1 + \frac{\tilde{q}/h}{\delta-\xi_1}\right] > 0,\ w_2 \equiv v'(\tilde{\psi})\left(1 - \frac{\gamma\tilde{\chi}}{\xi_1\tilde{\mu}}\right)\left[1 + \frac{\tilde{q}/h}{\delta-\xi_1}\right] > 0.$

3. Intertemporal Government Budget Constraint

Defining real government debt in terms of the domestic capital stock $\omega \equiv a/k$, we calculate the differential equation for ω as $\dot{\omega} = [\bar{g} - \bar{T}(t)\alpha + \left[r(\psi) - \frac{q-1}{h}\right]\omega - \tau\alpha\sigma \cdot \mu$. The basic intertemporal government budget constraint for the economy expressed as the present discounted value of taxes required to maintain intertemporal solvency of the public sector budget is then

$$V(T/k) \equiv \int_0^\infty [T(s)/k(s)]e^{-\delta s}ds = \omega_0 + \frac{(\bar{g}-\tau)\alpha}{\delta} - \frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma+\delta)\delta} - (\tilde{\psi} - \psi_0), \tag{12}$$

where $V(T/k)$ represents the measure of sustainable long-run fiscal balance. First, more initial stock of public debt (scaled by the capital stock) ω_0 increases the long term tax liability. Second, an increase in the primary deficit $(\bar{g} - \tau)$, represented by the fraction of output devoted to public expenditure minus the income tax rate, increases the tax liability as well. Thirdly, the term reflecting the inflation tax revenue, represented by the term $\frac{\sigma\gamma \cdot \tilde{\chi}}{(\sigma+\delta)\delta} = \frac{\sigma \cdot \tilde{\mu}}{\delta}$, decreases the tax liability since it is a source of revenue to the government. More importantly, in contrast to Bianconi and Fisher (2005), the tax liability depends upon the long-run accumulation of national debt in terms of the capital stock $(\tilde{\psi} - \psi_0)$, as well as on the speed of stable adjustment $\xi_1 < 0$. Hence, qualitatively, along the transitional path, depending on whether or not the long run national debt is above or below its initial level gives the effect of the transitional dynamics term on the tax liability. If the long run national debt is above its initial value, there is an increase in foreign debt along the transition and $(\tilde{\psi} - \psi_0) > 0 \Rightarrow -\frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0) < 0$, so that the tax liability is smaller *ceteris paribus.* Conversely, if the long run national debt is below its initial value, then $(\tilde{\psi} - \psi_0) < 0 \Rightarrow -\frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0) > 0$ and the tax liability is larger.

4. Balanced Growth Path, Long-Run Effects and Impact Effects

Letting $\dot{\chi} = \dot{\psi} = \dot{q} = 0$, the long-run equilibrium of the small open economy corresponds to three equations that determine the values of $\{\tilde{q}, \tilde{\psi}, \tilde{\chi}\}$ and as in Van der Ploeg (1996) and Turnovsky (1997), the endogenous adjustment of the national real interest rate $r(\tilde{\psi})$ insures that in long-run equilibrium the ratios of consumption and national debt to domestic capital reach their steady-state values and, thus, that the economy ultimately attains a common growth rate $\tilde{\phi}$. The policy parameters are $\{\bar{g}, \tau, \sigma; \omega(0)\}$ while the model parameters are $\{h, \delta, \alpha, \gamma, r(i^*, p^*, v)\}$.[6] In Table 1 we present steady state and impact effects of the fiscal and monetary policy and the parameter of the real interest function (including the risk premium relationship) on the balanced growth equilibrium evaluated at the initial equilibrium. The order of the effects (from left to right in Table 1) represents the order of impact on the overall economy. A shift in the capital tax rate affects all endogenous variables, followed by the real interest function, which, however, has no effect on growth $\tilde{\phi}$. Both of those exogenous factors do impact upon the long run national debt per unit of capital, $\tilde{\psi}$, and thus lead to transitional dynamics towards the stable adjustment path to the long

Table 1. Comparative Statics—Long Run and Impact Effects

	τ	r	$\bar{g}$	σ
$\tilde{q} \propto \tilde{\phi}$	$\frac{-h\alpha}{\tilde{q}+h\delta} < 0$	0	0	0
$\tilde{\psi}$	$\frac{-\alpha}{r'(\tilde{q}+h\delta)} < 0$	$\frac{-1}{r'} < 0$	0	0
$r(\tilde{\psi})$	$\frac{-\alpha}{\tilde{q}+h\delta} < 0$	0	0	0
$\tilde{\chi}$	$\frac{\alpha}{r'}\left[\frac{r'\tilde{q}+\delta}{\tilde{q}+h\delta}\right] > 0$	$\frac{\delta}{r'} > 0$	$-\alpha < 0$	0
$\tilde{\mu}$	$\left(\frac{\gamma}{\sigma+\delta}\right)\frac{\alpha}{r'}\left[\frac{r'\tilde{q}+\delta}{\tilde{q}+h\delta}\right] > 0$	$\frac{\gamma}{\sigma+\delta}\frac{\delta}{r'} > 0$	$\frac{-\gamma}{\sigma+\delta}\alpha < 0$	$-\frac{\gamma\cdot\tilde{\chi}}{(\sigma+\delta)^2} < 0$
$\tilde{\omega}$	$\frac{\alpha}{\delta}+\frac{\sigma\gamma}{(\sigma+\delta)\delta}\frac{\partial\tilde{\chi}}{\partial\tau}$	$\frac{1}{r'}\left(\frac{\sigma\delta}{\sigma+\delta}\right) > 0$	$-\frac{\alpha}{\delta}\left[1+\frac{\sigma\gamma}{\sigma+\delta}\right] < 0$	$\frac{\gamma\cdot\tilde{\chi}}{(\sigma+\delta)^2} > 0$
W	$\frac{\alpha}{v'}\left(\frac{1}{\tilde{q}+h\delta}\right)\left[\left(\frac{1+\gamma}{\delta\cdot\tilde{\chi}}\right)(v'\tilde{q}+\delta)-\left(\frac{w_1+\gamma\cdot w_2}{\delta-\xi_1}\right)\right]$	$\frac{1}{v'}\left[\left(\frac{1+\gamma}{\tilde{\chi}}\right)-\left(\frac{w_1+\gamma\cdot w_2}{\delta-\xi_1}\right)\right]$	$\frac{-\alpha}{\delta}\left(\frac{1+\gamma}{\tilde{\chi}}\right) < 0$	$\frac{-\gamma}{\delta(\sigma+\delta)} < 0$
$q(0)$	$\frac{-\alpha[\tilde{q}+h(\delta-\xi_1)]}{(\tilde{q}+h\delta)(\delta-\xi_1)} < 0$	$\frac{-\tilde{q}}{\delta-\xi_1} < 0$	0	0
$\chi(0)$	$\frac{\alpha}{\tilde{q}+h\delta}\left[\left(\tilde{q}+\frac{h}{v'}\right)+\frac{\tilde{\chi}[h\delta-\xi_1(h+\tilde{q})]}{h\xi_1(\delta-\xi_1)}\right]$	$\frac{\delta}{v'}+\frac{\tilde{\chi}}{\xi_1}\left[1+\frac{\tilde{q}}{h(\delta-\xi_1)}\right]$	$-\alpha < 0$	0
$\mu(0)$	$\frac{\gamma}{\sigma+\delta}\frac{\partial\tilde{\chi}}{\partial\tau}+\frac{\Omega}{(\sigma+\delta-\xi_1)}\frac{\partial\tilde{\psi}}{\partial\tau}$	$\frac{1}{r'}\left[\frac{\gamma\delta}{\sigma+\delta}-\frac{\Omega}{\sigma+\delta-\xi_1}\right]$	$\frac{-\gamma\alpha}{\sigma+\delta} < 0$	$-\frac{\gamma\cdot\tilde{\chi}}{(\sigma+\delta)^2} < 0$

run balanced growth path. This is followed by government spending and money growth which have no effect on growth.

The capital tax rate has a negative effect on long run growth and on the shadow value of capital. It also has a negative effect on the long run national debt and shifts resources to consumption and the accumulation of real money balances through a lower domestic price levels. The effect of the capital tax rate on the long run stock of government debt is ambiguous, because while there is the direct, negative, effect on long term growth that raises the stock of government debt, there is also negative indirect effect through additional consumption and money balances (the inflation tax channel). The real interest function discourages foreign borrowing thus decreases the long-run stock of foreign debt. This shift increases long run consumption and money balances and raises the long run government debt though the additional real interest cost of debt. Government spending reduces long term money balances through higher price levels thus reducing long term government debt; but higher money growth has the opposite effect on long term government debt because there is no direct long term consumption effect in this case. A change in the capital tax rate has two opposing effects on welfare. The positive effect of the tax rate on consumption and money balances raises welfare, while the negative effect on foreign debt, through the transitional dynamics, lowers welfare. Similarly for a change in the interest rate function. A change in government spending or in the rate of growth of money lowers welfare unambiguously through the consumption and money balances channel.

The impact effect of a change in the capital tax rate is unambiguously negative on the marginal cost of capital, but ambiguous for initial consumption and money balances. The reason is that there is a positive effect from higher long term consumption but a negative effect from lower long term foreign debt. An increase in the real interest function also decreases the initial marginal cost of capital but has similar ambiguous effects in initial consumption and money balances. A change in government spending does not affect the initial marginal cost of capital, and it crowds out initial private consumption and reduces initial money balances thus increasing the initial domestic price level. Higher money growth reduces initial money balances as well. From Table 1, it is clear that changes in government spending and money growth do not give rise to transitional dynamics, the economy jumps from one balanced growth path to another directly.

5. Budget Policies and Analysis

This paper adds an important link between fiscal policy and the current account, thus shedding light, among other things, on the twin deficits hypothesis. The key relationship that illustrates this issue is equation (12) which we rewrite as $\omega_0 + \frac{(\bar{g}-\tau)\alpha}{\delta} = V(T/k) + \frac{\sigma \cdot \tilde{\mu}}{\delta} + \frac{\Phi}{\delta - \xi_1} \cdot (\tilde{\psi} - \psi_0)$. We observe that, holding other factors constant, this model is Ricardian since public debt and lump-sum tax liabilities are perfectly correlated, and the primary fiscal deficit $\bar{g} - \tau$ (net of inflation tax revenues), and foreign debt accumulation $(\tilde{\psi} - \psi_0)$, tend to move in the same direction.

First, we examine the effects of a change in government policy parameters and the real interest function on the long term tax liability $V(T/k)$, all evaluated at a given initial equilibrium. A change in the capital income tax τ is obtained by evaluating (12) as follows:

$$\frac{\partial V(T/k)}{\partial \tau} = -\frac{\alpha}{\delta} - \frac{\sigma\gamma}{(\sigma+\delta)\delta}\cdot\frac{\partial\tilde{\chi}}{\partial\tau} - \frac{\Phi}{\delta-\xi_1}\cdot\frac{\partial\tilde{\psi}}{\partial\tau} = -\frac{\alpha}{\delta} - \frac{\sigma}{\delta}\cdot\frac{\partial\tilde{\mu}}{\partial\tau} - \frac{\Phi}{\delta-\xi_1}\cdot\frac{\partial\tilde{\psi}}{\partial\tau} \gtrless 0. \quad (13)$$

There are three distinct effects on the long-run tax liability $V(T/k)$. The first is the negative effect on the primary deficit, $-(\alpha/\delta)$, which decreases the long-run tax liability. The next term, $-\frac{\sigma\gamma}{(\sigma+\delta)\delta}\cdot\frac{\partial\tilde{\chi}}{\partial\tau} = -\frac{\sigma}{\delta}\cdot\frac{\partial\tilde{\mu}}{\partial\tau}$ captures the fact that an increase in the tax, because it discourages capital accumulation, increases the long-run consumption–capital ratio $\tilde{\chi}$, which, because it also results in an increase in the real money capital ratio $\tilde{\mu}$, increases inflation tax revenues and reduces $V(T/k)$. The last term $-\frac{\Phi}{\delta-\xi_1}\cdot\frac{\partial\tilde{\psi}}{\partial\tau}$ refers to the effect on national indebtedness and is part of the transitional dynamic adjustment induced by the tax change, but is of the opposite sign as the first two effects. An increase in the capital tax lowers the growth rate, which lowers $\tilde{\psi}$. This will increase the long run liability because lower growth lowers capital tax revenues. This is the main source of potential dynamic scoring in this paper.

A change in the share of government spending $\bar{g}$, evaluating (12) at the initial equilibrium $\frac{\partial C(T/k)}{\partial \bar{g}} = \frac{\alpha}{\delta}\left[1+\frac{\sigma\gamma\delta}{\sigma+\delta}\right] > 0$. In this case there are two direct effects and no transitional dynamics. The first is the positive effect on the primary deficit α/δ, which increases the long-run tax liability. The next term $\sigma\delta/(\sigma+\delta)$ captures the fact that an increase in the government spending crowds out private consumption and reduces money balances thus increasing $V(T/k)$. Both channels lead to an increase in future tax liabilities. Similarly, a change in the rate of growth of money σ, yields $\frac{\partial V(T/k)}{\partial\sigma} = \frac{\gamma\tilde{\chi}}{(\sigma+\delta)^2} < 0$. There are no transitional dynamics in this case as well and only the inflation tax effect. An increase in the growth of money reduces money balances (increases the price level), but at the initial equilibrium it increases inflation tax revenues and reduces $V(T/k)$.

Next, we consider a balanced-budget change in the capital tax, i.e. $d\tau = d\bar{g}$. We find

$$\begin{aligned}\frac{\partial V(T/k)}{\partial\tau}_{|d\tau=d\bar{g}} &= -\frac{\sigma}{\delta}\cdot\frac{\partial\tilde{\mu}}{\partial\tau}_{|d\tau=d\bar{g}} - \frac{\Phi}{\delta-\xi_1}\cdot\frac{\partial\tilde{\psi}}{\partial\tau}_{|d\tau=d\bar{g}} \\ &= \frac{\alpha}{r'(\tilde{\psi})(\tilde{q}+\delta h)}\left\{\frac{\Phi}{\delta-\xi_1} - \frac{\gamma\sigma[1-hr'(\tilde{\psi})]}{(\sigma+\delta)}\right\} \gtrless 0.\end{aligned}$$

Clearly, since the primary deficit is unaffected by the balanced budget tax cut, implications for long-run tax liabilities depend solely on the responses of $\tilde{\mu}$ and $\tilde{\psi}$. It is straightforward to show that the change in the consumption–capital ratio is ambiguous, since an (de)increase in government expenditure crowds-in(out) consumption. If the consumption–capital ratio rises (declines) on net, then so does the real money–capital ratio, which increases (decreases) inflation tax revenues and raises the possibility of dynamic scoring in response to a balanced-budget tax cut. In contrast, the response of national indebtedness is the same whether or not $\bar{g}$ falls along with τ, or $\frac{\partial\tilde{\psi}}{\partial\tau} = \frac{\partial\tilde{\psi}}{\partial\tau}_{|d\tau=d\bar{g}} = \frac{\alpha}{v'(\tilde{q}+\delta h)} < 0$. Thus, the rise (decline) in $\tilde{\psi}$ contributes to the rise(decline) in long-run tax liabilities $V(T/k)$. It follows then:

PROPOSITION 1. *A cut in the capital income tax rate decreases long term government liability if* $\frac{\alpha}{\delta}+\frac{\sigma}{\delta}\cdot\frac{\partial\tilde{\mu}}{\partial\tau}<-\frac{\Phi}{\delta-\xi_1}\cdot\frac{\partial\tilde{\psi}}{\partial\tau}$, *i.e. the overall weighted effect on foreign borrowing is larger than the direct effect plus the inflation tax effect; or equivalently* $r'(\tilde{\psi})<\frac{[\delta\phi/(\delta-\xi_1)]-\sigma\delta}{\tilde{q}(1+\sigma)+h\delta}$. *This result emerges directly from Table 1 and the condition on the slope of the premium function is obtained by direct substitution.*

PROPOSITION 2. *A necessary condition for a cut in the capital income tax rate balanced with a cut in the fraction of government spending to decrease the long term government liability is* $\frac{\Phi}{\delta-\xi_1}>\frac{\gamma\sigma[1-hr'(\tilde{\psi})]}{(\sigma+\delta)}$; *and a sufficient condition is that* $1-hr'(\tilde{\psi})<0\Leftrightarrow r'(\tilde{\psi})>1/h$. *This proposition is a direct consequence of Table 1 as well.*

Note that in both propositions we identify the region of the slope of the risk premium function that allows for dynamic scoring. In Proposition 1, the slope cannot be large because the cut in capital income tax at home has to encourage borrowing abroad to further increase economic growth. Hence, dynamic scoring only occurs if the condition of the supply of debt is satisfied. In Proposition 2 because government spending is adjusting as well, the sufficient condition implies that the slope of the premium function has to be greater than the inverse of the slope of the investment adjustment cost parameter. As long as the slope of the premium function is high enough to discourage a relatively high level of foreign borrowing, the likelihood of dynamic scoring is increased.

We now turn to the issue of the long run sustainability of budget policies. Fiscal and monetary policies are set so no future tax liabilities are needed to balance the intertemporal budget.[7] This can be guaranteed by the choice of one (or more) of the government policy parameters $\{\bar{g},\tau,\sigma;\omega(0)\}$ under the restriction that $V(T/k)=0$. In the next section we resort to numerical simulations.

6. Numerical Simulations

We provide numerical evaluations given the nonlinearities of the model equilibrium. The benchmark set of parameter values are $h=1$, $\delta=0.04$, $\alpha=0.1$, $\tau=0.31$, $\bar{g}=0.11$, $\gamma=0.20$, $\sigma=0.04$, $k_0=10$ (so that $\alpha k_0=1$), $b_0=0.50$, $i^*=0.10$, $p^*=0.04$, $a_0=0.55$, $z_0=0.05$, where the implied value of Tobin's q is $1.0275>1$ so that the equilibrium is characterized by 2.75% on-going growth. The interest premium convex function is $v(\tilde{\psi})=s_1\exp(s_2+s_3\tilde{\psi})$, $s_1,s_2,s_3>0$, with $s_1=0.00002$, $s_2=5.75$ and $s_3=1.75$. The initial equilibrium is one where the tax rate is large relative to the government share of output. This initial equilibrium implies that the initial tax liability, $V(T/k)$ is 0.025, or 2.5% of output; the real interest rate from the interest rate function is 6.75%; the national debt of the nation is about 10% of output (the country is a net debtor to the rest of the world); and the half life to the balanced growth path is about 6.67 periods.

Table 2 presents comparative statics effects of government policy changes on the long term liability of the government and welfare evaluated at the initial equilibrium. The first two columns refer to the base parameter set while the last two columns refer to the case where the interest premium function is steeper, i.e. s_3 increases to 2.75. An increase in the capital income tax does not have Laffer style effects if the premium

Table 2. Policy changes from an initial equilibrium

	Base parameter set		*Sensitivity: s3 increases to 2.75*	
	$\partial V(T/K)$ *% from initial eq.*	∂W *% from initial eq.*	$\partial V(T/K)$ *% from initial eq.*	∂W *% from initial eq.*
$\partial\tau > 0$; $\tau = 0.36$(*)	**200.4**	**0.020**	**–9.9**	**0.015**
$\partial\bar{g} > 0$; $\bar{g} = 0.16$	**510.4**	**–3.280**	**304.3**	**–3.200**
$\partial\sigma > 0$; $\sigma = 0.04125$	**–8.9**	**–0.097**	**–5.4**	**–0.097**
$\partial r > 0$; $r = 0.073$(*)	**665.3**	**9.980**	**276.9**	**6.180**
$\partial\tau = \partial\bar{g} > 0$;(*) $\tau = 0.36$ $\bar{g} = 0.16$	**717.6**	**–2.740**	**299.4**	**–2.790**

Note: (*) indicates the policy change generates transitional dynamics.

function is steeper, e.g. Proposition 1. The second row is an increase in the government share of GDP and at the higher slope of the interest rate function, borrowing abroad is more costly, this mitigates the increase in liability and the loss in welfare. The case of an increase in the rate of growth of money decreases the future tax liability and welfare though the impact on lower real money balances (inflation tax effect). The case of a higher real interest rate in Table 2 refers to either an increase in the foreign interest rate or a decrease in foreign inflation; in this case there is no change in the slope of the premium function. The long term liability increases dramatically and welfare also increases in the base set. The next row shows the effect of increase in the government share of GDP financed by higher capital taxes. This has the largest impact in the long term liability among all the policies considered and a moderate negative impact on welfare. The steeper interest premium function makes this effect on the long term liability much smaller because the dynamic scoring effect of the capital tax rate disappears, a result consistent with Proposition 2. The effect on welfare is negative and very similar in both cases.

Table 3 presents results referring to policies that satisfy the long term constraint that tax liabilities are zero, or $V(T/k) = 0$. When the policy change leads to transitional dynamics, in order to obtain numerical evaluations of policies that guarantee intertemporal sustainability, we need to evaluate the parameter that solves (12) for $V(T/k) = 0$. We use a simple shooting algorithm, e.g. Judd (1998). This process iterates until convergence is obtained. When policy change does not lead to transitional dynamics, the change occurs instantaneously.

The first row of Table 3 shows the case for the capital income tax rate as a single policy instrument. At the base parameter set, the capital income tax rate should be reduced by approximately 0.031 percentage points, with convergence achieved in four iterations and 10E-6 accuracy. This is obtained through the dynamic scoring effect under high tax/low spending levels initially, and a welfare loss of about 0.33% occurs. However, the convergence of the capital income tax instrument is not robust. At the steeper interest rate function, the capital income tax alone cannot achieve long term budget balance since there is no convergence. Figure 1 illustrates the problem of capital tax finance of the long-term liability for the steeper interest rate function. The

Table 3. Policy changes to balance long term liability: $V(T/K) = 0$

Base Parameter Set				*Sensitivity: s3 increases to 2.75*			
	# of iterat.	*Accur.*	*∂W % from initial eq.*		*# of iterat.*	*Accur.*	*∂W % from initial eq.*
$\partial\tau < 0$; $\tau = 0.0279$**(*)**	**4**	**10E-6**	**–0.337**	$\partial\tau$**(*)**	**NoConv.**	**—**	**—**
$\partial\bar{g} < 0$;	**—**	**—**	**0.656**	$\partial\bar{g} < 0$;	**—**	**—**	**0.517**
$\bar{g} = 0.090$				$\bar{g} = 0.093$			
$\partial\tau = \partial\bar{g} < 0$; **(*)**	**4**	**10E-7**	**0.264**	$\partial\tau = \partial \cdot \bar{g} < 0$;**(*)**	**4**	**10E-7**	**0.281**
$\tau = 0.303$				$\tau = 0.290$			
$\bar{g} = 0.103$				$\bar{g} = 0.090$			

Note: (*) indicates the policy change generates transitional dynamics.

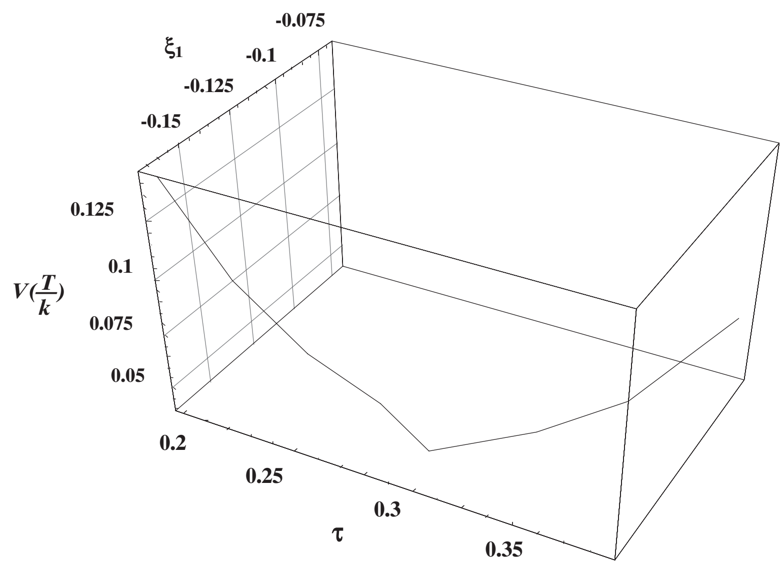

Figure 1. Capital Income Tax under Long Term Balance and More Inelastic Supply of Debt (Steep Interest Premium Function): $s_3 = 2.75$; $V(T/k) = 0$

vertical axis is the long term liability, the lower axis is the tax rate and the upper axis is the stable root that determines the speed of adjustment to the long term growth path. It is clear that at alternative levels of the tax rate, the long term liability does not get to the lower zero bound. At low tax rates, the root declines and the long term liability also declines, but close to the 30% tax rate, this process reverses and the long term liability and the stable root increase. The next row of Table 3 shows the case in which government spending adjusts to balance the intertemporal budget and an instantaneous decrease of 0.02 percentage points from the initial level of government spending takes the economy to the new balanced growth path such that $V(T/k) = 0$. The steeper interest premium function requires a slightly smaller decrease in government spending. In both cases, the welfare gains are small but robust because of the additional private consumption and real money balances. Finally, we consider the case where both the tax rate and government spending are used to balance the intertemporal budget. This policy converges in both interest premium functions. The decrease in government spending and taxes is very small under the base set and slightly larger under the alternative interest premium, while the welfare gains are robust for both cases. The combined tax-cum-spending mix does achieve long term balance with four iterations and plausible accuracy.

7. Concluding Remarks

Our novel result is that the effects of fiscal policies, in particular a cut in capital income tax, depend on its spillover on the economy national foreign borrowing. A decrease in the capital tax increases the growth rate, which increases foreign debt; this will decrease the long run liability because higher growth increases capital tax revenues and reduces the long term tax liability. However, under unitary elasticity of substitution in utility, this effect depends critically upon the elasticity of supply of debt represented by the slope of the interest premium function. If the distortion is too large (the slope is too steep), the needed foreign borrowing is curtailed and growth is not enough to generate the revenues needed for dynamic scoring. Hence, in our framework, foreign deficits are negatively related to the long term liability of the government. This effect is enlarged in the case where a tax-cum-expenditure policy is used. Our simulations confirm that using capital income taxes alone to balance the intertemporal government budget constraint may not be feasible if the slope of the interest premium function is too steep. Intertemporal balance can, however, be achieved with a tax-cum-expenditure policy or government expenditure policy changes alone under more restricted capital flows.

There are several potential avenues for future research such as extending to a more general class of utility, such as the constant elasticity case; and at the empirical dimension, testing further the hypotheses of the relationship between the current account and the intertemporal government budget constraint would be very fruitful.

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Notes

1. E.g. Ireland (1994), Bruce and Turnovsky (1999), Agell and Persson (2001), Novales and Ruiz (2002), Mankiw and Weinzerl (2006), Leeper and Yang (2008) and Trabandt and Uhlig (2011).

2. Chen et al. (2008) examines a related problem from the perspective of Taylor rules for interest rates.

3. E.g. Ireland (1994), Bruce and Turnovsky (1999), Bianconi (1999), Agell and Persson (2001), Novales and Ruiz (2002), Bianconi and Fisher (2005), Mankiw and Weinzerl (2006), Leeper and Yang (2008) and Trabandt and Uhlig (2011).

4. For the twin deficits see e.g. Chinn (2005) and Gagnon (2011). Related to the current debt crisis of small open economies in Europe such as Greece, Portugal, Ireland and Cyprus more recently; see e.g. Cavallari and Gioacchino (2005), Colciago et al. (2008) and Alesina and Ardagna (2010). On the effects of inflation see Engen and Hubbard (2004), Aizenman and Marion (2011) and Hall and Sargent (2011).

5. See e.g. Jones and Manuelli (1990), Turnovsky (1996); and Hayashi (1982) for the Tobin's q model of investment.
6. The initial stock of public debt is included as a policy parameter for the potential case of a change in government policy parameters financed by a swap with initial public debt; see e.g. Novales and Ruiz (2002).
7. See also Agell and Persson (2001) and Fredriksson (2007). Ostry et al. (2010) introduce an alternative concept of 'fiscal space' in reference to the difference between the stock of debt limit (requiring intertemporal balance) and the stock of current debt.